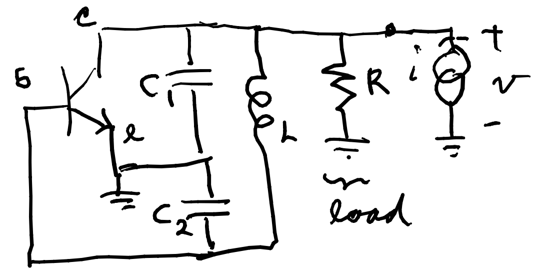
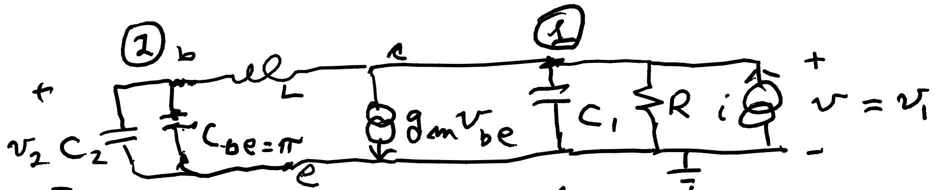
Colpitts

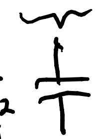
EE303
03/12/09



look at input impedance
then set $i = 0$

$v = z_{in} \cdot i$ if $i = 0$ & $v \neq 0$
then $z_{in} = \infty$





$\hat{C}_2 = C_2 + C_\pi$

node equations KCL

① : $0 = i - G v_1 - sC_1 v_1 - g_m v_2 + \frac{1}{sL}(v_2 - v_1)$

② : $0 = -s\hat{C}_2 v_2 - \frac{1}{sL}(v_2 - v_1)$

$$\begin{bmatrix} i \\ 0 \end{bmatrix} = \begin{bmatrix} G + sC_1 + \frac{1}{sL} & g_m - \frac{1}{sL} \\ -\frac{1}{sL} & s\hat{C}_2 + \frac{1}{sL} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\left(s\hat{C}_2 + \frac{1}{sL}\right) v_2 = \frac{1}{sL} v_1 \Rightarrow v_2 = \frac{1}{s^2 L\hat{C}_2 + 1} \cdot v_1$$

$$i = \left[G + sC_1 + \frac{1}{sL} + \left(g_m - \frac{1}{sL}\right) \cdot \frac{1}{s^2 L\hat{C}_2 + 1}\right] v_1$$

$$v = \frac{s^2 L\hat{C}_2 + 1}{(s^2 L\hat{C}_2 + 1)\left(G + sC_1 + \frac{1}{sL}\right) + g_m - \frac{1}{sL}} \cdot i$$

$$z_{in}(s) = \frac{sL(s^2 L\hat{C}_2 + 1)}{(s^2 L\hat{C}_2 + 1)(s^2 C_1 L + sLG + 1) + g_m sL - 1} = \frac{N(s)}{D(s)}$$

seen at output of Colpitts

$\Rightarrow$ zeros over poles

here s cancels numerator & denominator

$$= \frac{sL(s^2 L\hat{C}_2 + 1)}{s^4 L^2 C_1 \hat{C}_2 + s^3 L^2 \hat{C}_2 G + s^2 L\hat{C}_2 + s^2 C_1 L + sLG + \cancel{1} + g_m sL \cancel{- 1}}$$

$$= \frac{L(s^2 L\hat{C}_2 + 1)}{s^3 L^2 C_1 \hat{C}_2 + L^2 \hat{C}_2 G s^2 + L(\hat{C}_2 + C_1)s + L(G + g_m)}$$

as desire an oscillator; as $v \neq 0$, $i = 0 \Rightarrow$ denominator $\Rightarrow 0$ of $Z_{in}(s)$

then desire $D(j\omega) = 0$ ie $s = j\omega$

$$D(s) = LC_1\hat{C}_2 s^3 + L\hat{C}_2 G s^2 + (C_1+\hat{C}_2)s + (C+g_m)$$

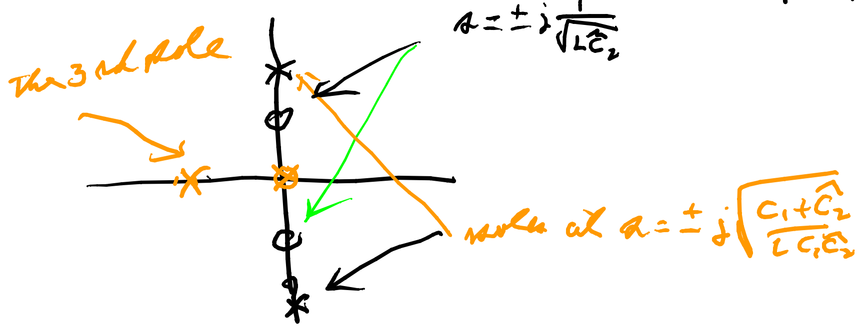
$$D(j\omega) = -j\omega^3 LC_1\hat{C}_2 - L\hat{C}_2 G\omega^2 + j\omega(C_1+\hat{C}_2) + (G+g_m) = 0$$

$$= j\omega\Big[\underbrace{-\omega^2 LC_1\hat{C}_2 + [C_1+\hat{C}_2]}_{=0}\Big] + \Big[\underbrace{G+g_m - L\hat{C}_2 G\omega^2}_{=0}\Big]$$

$\omega^2 = \dfrac{C_1+\hat{C}_2}{LC_1\hat{C}_2} \Rightarrow$ this is the parallel combination of C's resonating with L

$$G+g_m = L\hat{C}_2 G\omega^2 = G\frac{C_1+\hat{C}_2}{C_1} \Rightarrow g_m = G\Big[-1+1+\frac{\hat{C}_2}{C_1}\Big] = G\frac{\hat{C}_2}{C_1}$$

now this will oscillate at $\omega_0 = 2\pi f = \sqrt{\dfrac{C_1+\hat{C}_2}{LC_1\hat{C}_2}}$

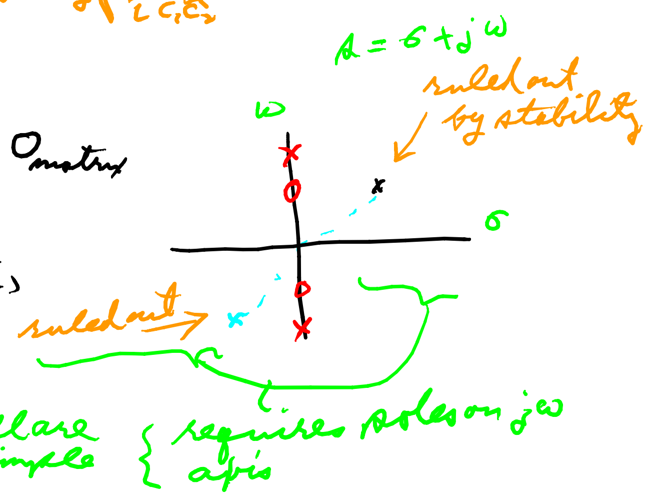


For lossless: $Y(s) + Y^T(-s) = 0_{matrix}$

for a 1-port $y(s) = -y(-s)$

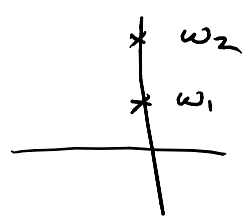
then also $\dfrac{1}{z(s)} = -\dfrac{1}{z(-s)}$ for $z(s) = \dfrac{1}{y(s)}$

$\Rightarrow z(s) = -z(-s)$



$$y(s) = \frac{k_1 s}{s^2+\omega_1^2} + \frac{k_2 s}{s^2+\omega_2^2} + \cdots$$

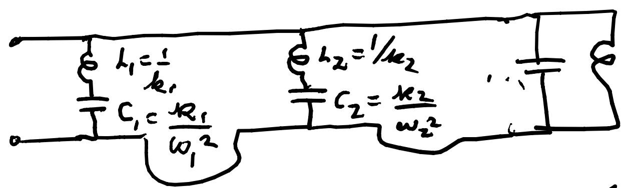
allows the circuit to be tuned to several frequencies



$$y_1(s) = \frac{k_1 s}{s^2+\omega_1^2} = \frac{1}{\frac{1}{k_1}s + \frac{\omega_1^2}{k_1 s}} = \frac{1}{z_1(s)} \Rightarrow z_1(s) = \frac{1}{k_1}s + \frac{\omega_1^2}{k_1 s}$$

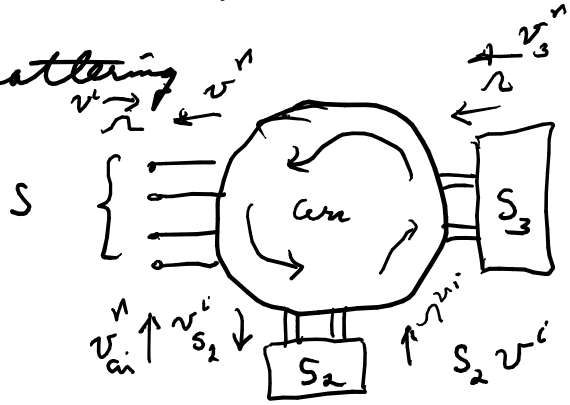
$\Rightarrow$ series LC: $L = \frac{1}{k_1}$, $C = \frac{k_1}{\omega_1^2}$

$y(s) \Longrightarrow$



∴ can synthesize a lossless 1-port by such circuit

For scattering



S_i is 2×2

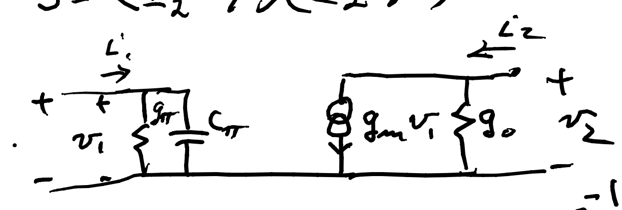
$$v^r = S_3 S_2 v^i = S\cdot v^i$$

$$S = S_2 S_3$$

$$S_{cn} = \left[\begin{array}{cc|cc|cc} 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ \hline 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ \hline 0&0&1&0&0&0\\ 0&0&0&1&0&0 \end{array}\right]$$

For the BJT: to find S

$$S = (1_2 - Y)(1_2 + Y)^{-1} = (1_2+Y)^{-1}(1_2 - Y)$$

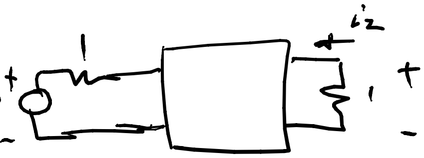


$$Y = \begin{bmatrix} \overset{= g_\pi}{g_\pi + sC_\pi} & 0 \\ g_m & g_o \end{bmatrix}$$

$$S = \begin{bmatrix} 1-y_\pi & 0 \\ -g_m & 1-g_o \end{bmatrix}\begin{bmatrix} 1+y_\pi & 0 \\ g_m & 1+g_o \end{bmatrix}^{-1} = \begin{bmatrix} 1-g_\pi & 0 \\ -g_m & 1-g_o \end{bmatrix}\begin{bmatrix} \frac{1}{1+y_\pi} & 0 \\ \frac{-g_m}{(1+y_\pi)(1+g_o)} & \frac{1}{1+g_o} \end{bmatrix}$$

$$= \begin{bmatrix} s_\pi & 0 \\ \frac{-g_m(1+g_o) - g_m(1-g_o)}{(1+y_\pi)(1+g_o)} & s_o \end{bmatrix} = \begin{bmatrix} s_\pi & 0 \\ \frac{-2g_m}{(1+y_\pi)(1+g_o)} & s_o \end{bmatrix} = \begin{bmatrix} s_\pi & 0 \\ s_{21} & s_o \end{bmatrix}$$

$v_1 + i_1 = e$



$= 2v_1^i$

$v_2 = -i_2 \Rightarrow v_2 - i_2 = 2v^r = 2v_2$

$$\frac{2v_2}{e} = \frac{2v_2^r}{2v_1^i} = A_{21} \Rightarrow \frac{v_2}{e} = \frac{A_{21}}{2}$$

$\therefore$ for

$v \Rightarrow \dfrac{v_2}{e} = \dfrac{-g_m}{(1+g_\pi)(1+g_0)}$
