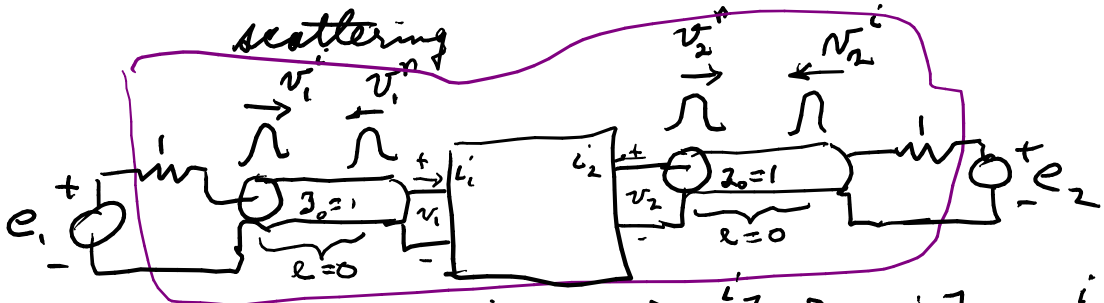
EE303
03/10/09

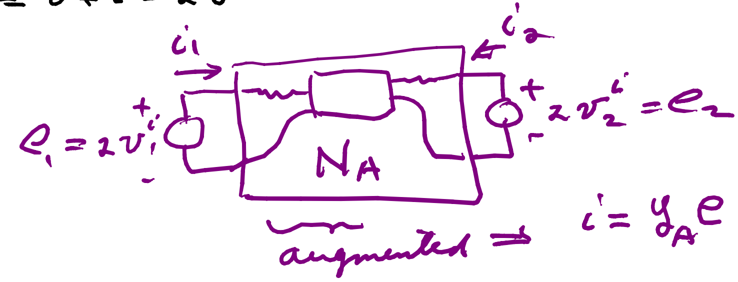


$$2v^r = 2\begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix} = \begin{bmatrix} v_1 - i_1 \\ v_2 - i_2 \end{bmatrix}, \quad 2\begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} = \begin{bmatrix} v_1 + i_1 \\ v_2 + i_2 \end{bmatrix} = 2v^i$$

$$v^r = S v^i$$

a network analyzer measures these v^i, v^r

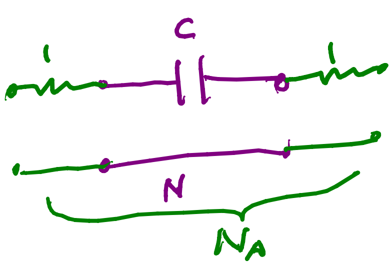
$$e = v + i = 2v^i$$



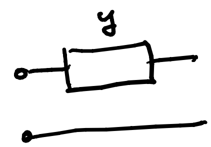
$\Rightarrow \quad i = y_A e$

$$i = y_A e$$
$$v = e - i = e - y_A e = (1_2 - y_A)\cdot e$$
$$v - i = 2v^r = (1_2 - y_A)e - y_A e = (1_2 - 2y_A)e$$
$$= (1_2 - 2y_A)\cdot 2v^i$$
$$v^r = S v^i$$
$$= (1_2 - 2y_A)\cdot v^i \Rightarrow S = 1_2 - 2y_A$$

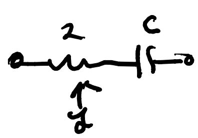
Ex:



To find y_A



here

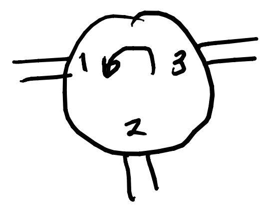


$$y = \frac{1}{z} = \frac{1}{2 + \frac{1}{Cs}} = \frac{Cs}{2Cs+1}$$

$$Y_A = \begin{bmatrix} y & -y \\ -y & y \end{bmatrix} = \frac{ca}{2ca+1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$ a singular matrix (no impedance matrix)

$$S = 1_2 - 2Y_A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{ca}{2ca+1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{ca+1}{2ca+1} & +\frac{ca}{2ac+1} \\ \frac{ca}{2ca+1} & \frac{ca+1}{2ca+1} \end{bmatrix}$$

Circulator



$$v_2^r = v_1^i$$
$$v_3^r = v_2^i$$
$$v_1^r = v_3^i$$

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

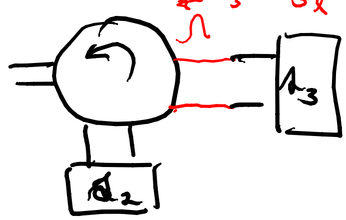
$v_{2\ell}^r = A_2 v_{2\ell}^i$ $\quad$ ℓ = load but $v_{2\ell}^i = v_2^r$ (see below)

$$v_3^i = v_{3\ell}^r = A_3 v_{3\ell}^i = A_3 v_3^r = A_3 v_2^i$$
$$= v_1^r$$
$$v_2^i = v_{2\ell}^r = A_2 v_{2\ell}^i = A_2 v_2^r = A_2 v_1^i$$

$$v_1^r = S v_1^i \qquad v_1^r = v_3^i = A_3 v_2^i = A_3 \cdot A_2 v_2^r$$
$$= A_3 A_2 v_1^i \Rightarrow S = A_3 \cdot A_2$$

$\leftarrow v_3^i = v_{3\ell}^r$

$S = A_3 \cdot A_2 \rightarrow$

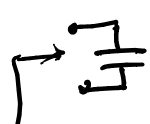


allows design

$$2v^i = v + i = v + yv = (1+y)v$$
$$2v^r = v - i = v - yv = (1-y)v$$

$$v^r = S v^i = ((1-y)v/2) = S(1+y)v/2$$

$$\Longrightarrow \quad S = \frac{1-y}{1+y}$$



$y = ac$

$$S = \frac{1-ac}{1+ac}$$

$$S^T(-a) S(a) = 1_2 \Leftarrow S(a) = \frac{1}{S(-a)} \text{ here}$$

(lossless condition)

Hurwitz conjugate

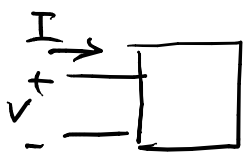
n

average Power in: $P(j\omega) = \mathrm{Re}\, V^{T*} I$

$$= \frac{V^{T*}I + V^{T}I^{*}}{2} = \frac{V^{T*}I + I^{T*}V}{2}$$

$I = Y(j\omega)V$

$$= \frac{V^{T*}Y(j\omega)V + V^{T*}Y^{T*}(j\omega)V}{2}$$



$$= V^{T*}\left(Y(j\omega) + Y^{T*}(j\omega)\right)V/2$$

can apply any V

if lossless $P(j\omega) = 0$ $\Rightarrow$ $Y(j\omega) + Y^{T*}(j\omega) = 0_n$

$\Rightarrow Y(j\omega) + Y^{T}(-j\omega) = 0_n$ } real components

$\Rightarrow Y(s) + Y^{T}(-s) = 0$ for $s = j\omega$

but if have real elements have analytic functions

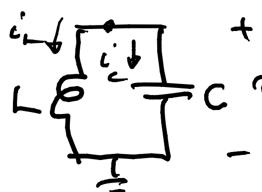
analytic continuation $Y(s) = -Y^{T}(-s)$ for all s, $s = \sigma + j\omega$

lossless condition

Ex: $\dashv\vdash$ $y = Cs$, $y(-s) = -Cs = -y(s)$

$S = (1_n - Y)(1_n + Y)$ $\Rightarrow$ for lossless $S^{T}(-s)S(s) = 1_n$

oscillators



$i_C(s) = sC \cdot v(s)$

$v(s) = sL \cdot i_L(s)$ $\quad i_L = -i_C$ by KCL

$i_C(s) = sCv(s) = -\frac{v(s)}{sL}$

$\Rightarrow s^2 LC\, v(s) + v(s) = 0$

$$LC \frac{d^2}{dt^2} v(t) + v(t) = 0$$

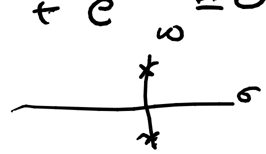
needs $v(0)$, $v'(0)$

has a different solution (for $t \to \infty$) with different initial.

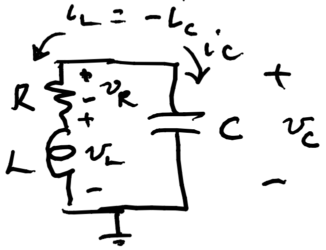
note: e^{st} satisfies this differential equation for $-\infty < t < \infty$

$$s^2 LC e^{st} + e^{st} = 0 \Rightarrow s^2 LC + 1 = 0$$

$$s^2 = -\frac{1}{LC} \Rightarrow s_{1,2} = \frac{\pm j}{\sqrt{LC}} = \pm j\omega_0$$



Interested in a structurally stable oscillator which this LC one is not; also due to losses in practice the oscillations die out



$i_L = -i_c$

$i_c = s C v_C$

$v_L = s L i_L = sL(-i_c)$, $v_R = R i_L = R(-i_c)$

$$v_L = -sL \cdot sC v_C = v_C - v_R = v_C + R s C v_C$$

$$(s^2 LC + RCs + 1) v_C = 0$$

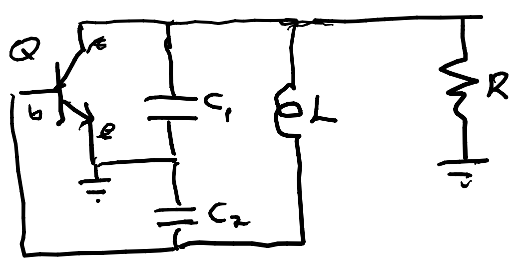
$$\left(s^2 + \frac{R}{L} s + \frac{1}{LC}\right) \cdot v_C = 0 \; ; \quad P(s) = s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + \frac{\omega_0}{Q} s + \omega_0^2$$

when $R = 0 \Rightarrow$ lossless $\Rightarrow Q = \infty$

need to turn to electronic circuits to get $Q = \infty$

p. 1174 = RC phase shift oscillator $\Rightarrow$ 3rd order to oscillate

p. 1179 = Colpitts $\Rightarrow$ also 3rd good for generating chaotic signals



$\Rightarrow$ equivalent

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for exam

load lines – diodes

Q pts.

BJT & MOS models

OTA's & diff. eqs

current mirrors