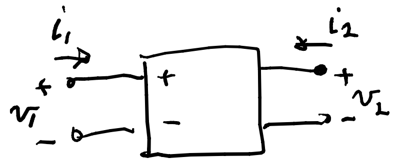
EE 303 H
02/26/09



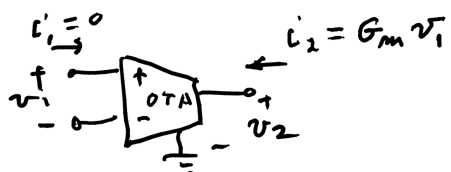
a 2-port

$$\Rightarrow \; i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$i = Yv \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

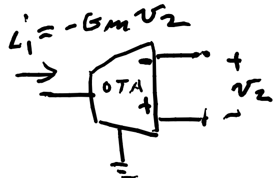
; $Y \Rightarrow$ 2x2 admittance matrix — describes a linear circuit

$$i_1 = y_{11} v_1 + y_{12} v_2$$
$$i_2 = y_{21} v_1 + y_{22} v_2$$

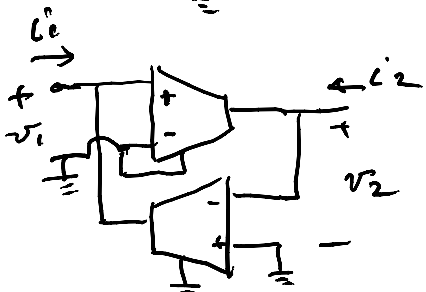


$$\Rightarrow \; i_1 = 0 \cdot v_1 + 0 \cdot v_2$$
$$i_2 = G_m v_1 + 0 \cdot v_2$$

$$\Rightarrow \; Y_{OTA} = \begin{bmatrix} 0 & 0 \\ G_m & 0 \end{bmatrix}$$



$$Y'_{OTA} = \begin{bmatrix} 0 & -G_m \\ 0 & 0 \end{bmatrix} = Y_{OTA}^{\text{Transpose}}$$



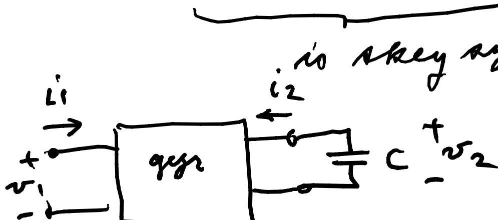
here v's are the same for OTA & OTA' & the currents add

$$Y_{OTA} v + Y'_{OTA} v = i$$

$$i = (Y_{OTA} + Y'_{OTA}) v$$

$$Y = \begin{bmatrix} 0 & -G_m \\ G_m & 0 \end{bmatrix} = -Y^T \quad T = \text{transpose} \quad \Rightarrow \text{a gyrator}$$

is skew symmetric



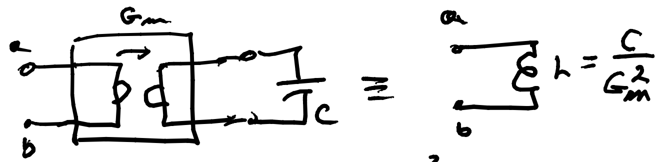
1) $i_2 = -sCv_2 = v_1 G_m + 0 \cdot v_2$

2) $i_1 = -G_m v_2 \Rightarrow v_2 = -\frac{1}{G_m} i_1$

from 1) $v_1 = -\frac{sC}{G_m} v_2 = -\frac{sC}{G_m} \cdot \left(-\frac{1}{G_m}\right) i_1$

$$v_1 = \left(s \frac{C}{G_m^2}\right) i_1$$

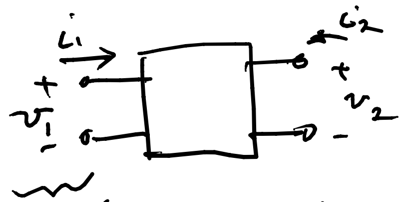
$\Rightarrow$



if $C = 1\,\mu Fd$, $G_m = \frac{1}{10^3}\,\mho = 10^{-3}\,\mho \Rightarrow L = \underbrace{10^6}_{1/G_m^2} \cdot \underbrace{10^{-6}}_{C} = 1\,Hy$

will 12 transistors for 2 OTA + 4 for I_T = 16 transistors & 1 cap.

$\Rightarrow$ a large inductor but not for large power.



$P_{\text{into port 1}} = v_1(t) \cdot i_1(t)$ $\quad$ $P_{\text{into port 2}} = v_2(t)\, i_2(t)$ $\Big\}$ $P_{\text{in 2-port}}(t) = v_1(t)\, i_1(t) + v_2(t)\, i_2(t)$

$= v^T(t)\, i(t) = P_{in}(t)$

for the gyrator

$$P_{in} = v^T \cdot i = v^T(t) \cdot Y \cdot v(t)$$

$$= [v_1 \quad v_2] \begin{bmatrix} 0 & -G_m \\ G_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= [0 \cdot v_1 + G_m v_2 \,,\; -G_m v_1 + 0 \cdot v_2] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

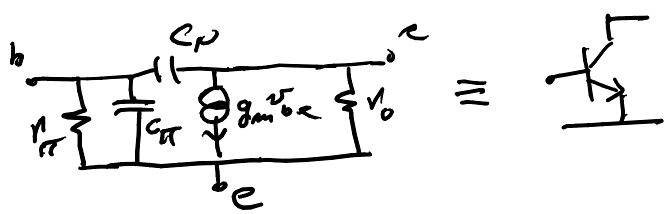
$$= [G_m v_1 v_2 - G_m v_1 v_2]$$

$$= 0$$

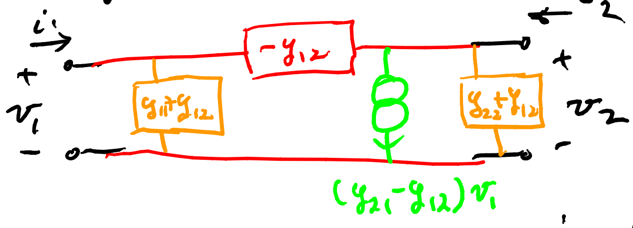
$\therefore$ signal power into a gyrator is zero $\}$ is lossless ($P_{in} \equiv 0$)

[for operation we have to bias the transistor & have nonzero bias current in I_T]

For the BJT:



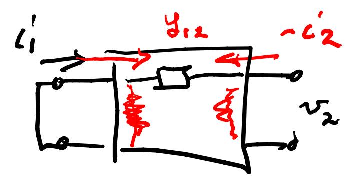
For Y's in general $i = Yv$



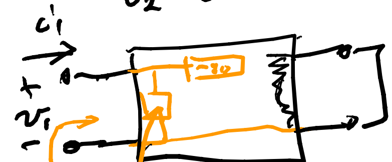
⇒ equivalent circuit for a 2-port with a Y matrix

$i_1 = y_{11}v_1 + y_{12}v_2$ $\qquad$ $i_2 = y_{21}v_1 + y_{22}v_2$

to find $y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1 = 0 = \text{a short}}$ $\qquad$ apply a v_2 & look for i_1



$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2 = 0 = \text{a short}}$

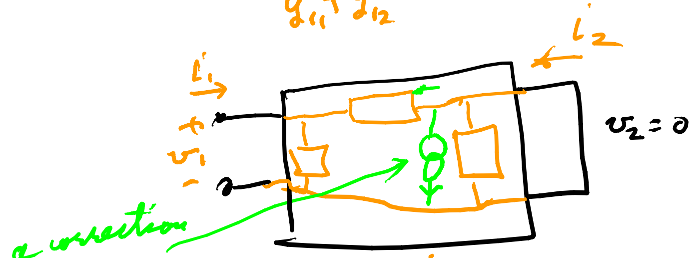


input admittance $y_{11} =$ || of two 1-ports

$y_{11} + y_{12}$

for y_{21}: $i_2 = y_{21}v_1 + y_{22}v_2$ (set = 0 by $v_2 = 0$)

$$\Rightarrow \left.\frac{i_2}{v_1}\right|_{v_2=0} = y_{21}$$



gives $\frac{i_2}{v_1} = y_{12} + (y_{21} - y_{12})$

a common description for 2-ports is the scattering matrix

$$\begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} = v^i \qquad v^r = \begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix}$$

$2v^i = v + Z_0 i$ $\qquad$ usually normalize $Z_0 = 1$

$2v^r = v - Z_0 i$ $\qquad$ Z_0 = characteristic

$$\Longrightarrow \left.\begin{aligned} 2v^i &= v + i \\ 2v^r &= v - i \end{aligned}\right\} \; v^r = S \cdot v^i$$