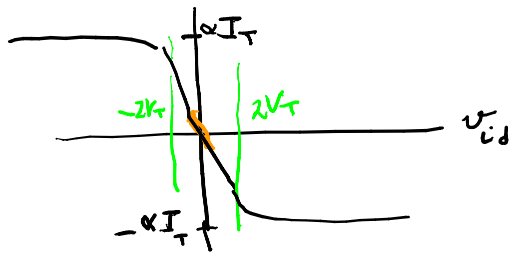
EE303H
02/24/09

BJT OTA

from last time

$$i_{out} = -\alpha I_T \tanh\left(\frac{v_{id}}{2V_T}\right)$$



$$\frac{di_{out}}{dv_{id}} = G_m$$

$$= -\alpha I_T \frac{d\left(\tanh(v_{id}/2v_T)\right)}{d v_{id}}$$

$$= -\alpha I_T \left(\frac{1}{2V_T}\right)\left[1-\tanh^2\left(\frac{v_{id}}{2V_T}\right)\right]$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{d\tanh x}{dx} = \frac{e^x-(-e^{-x})}{e^x+e^{-x}} - \frac{(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$= 1 - \frac{e^x-e^{-x}}{e^x+e^{-x}} = 1-\tanh x$$

$$G_m = -\frac{\alpha}{2}\frac{I_T}{V_T}\left(1-\tanh\left(\frac{v_{id}}{2V_T}\right)\right)$$

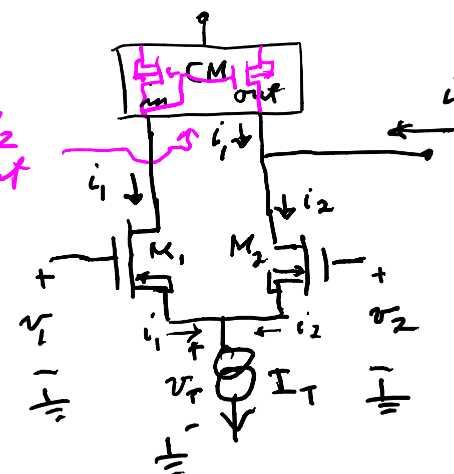
$\Rightarrow$ | @ $v_{id}=0$ $= -\alpha I_T/2V_T$

valid for linear region which is small input v_{id}

$$-2V_T < v_{id} < 2V_T$$

$V_T = 0.026V$ @ room Temp

MOS OTA via differential pair

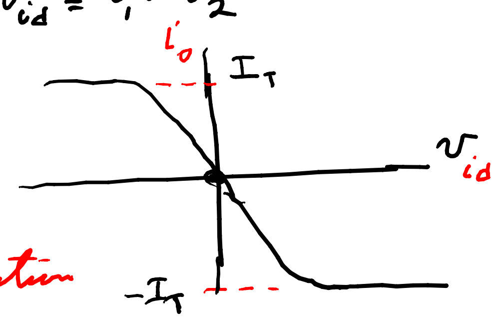


M_1 & M_2 see different transistor loads

$i_0 + i_1 - i_2 = 0$ KCL

$i_0 = i_2 - i_1$ & $I_T = i_1 + i_2$

$v_{id} = v_1 - v_2$



1) $i_0 = i_2 - i_1$

2) $I_T = i_2 + i_1$

3) $v_{id} = v_1 - v_2$

assume $M_1 = M_2$ in saturation

4) $i_1 = \frac{KP}{2}\frac{W}{L}(v_{GS_1} - VTO)^2$

5) $i_2 = \frac{KP}{2}\frac{W}{L}(v_{GS_2} - VTO)^2$

6) $v_{GS_1} = v_1 - v_T$

7) $v_{GS_2} = v_2 - v_T$

8) $v_{GS_1} - v_{GS_2} = v_1 - v_2 = v_{id}$

9) $$\frac{\sqrt{i_1} - \sqrt{i_2}}{\sqrt{\frac{KP}{2}\frac{W}{L}}} = (v_{GS_1} - \cancel{VTO}) - (v_{GS_2} - \cancel{VTO}) = v_{GS_1} - v_{GS_2} = v_{id}$$

but $i_2 = i_0 + i_1$ from (1) $\Rightarrow$ 9)

10) $\sqrt{i_1} - \sqrt{i_0 + i_1} = \sqrt{\frac{KP}{2}\frac{W}{L}}\, v_{id}$ $\Rightarrow$ square this

$$i_1 - 2\sqrt{i_0 + i_1}\cdot\sqrt{i_1} + i_0 + i_1 = \frac{KP}{2}\frac{W}{L} v_{id}^2$$

square again

$$4(i_0 + i_1)\, i_1 = \left(\frac{KP}{2}\frac{W}{L}\right)^2 v_{id}^4$$

instead square 9): $\underbrace{i_1 + i_2}_{I_T} - 2\sqrt{i_1 \underset{i_2 = i_0 + i_1}{i_2}} = \left(\frac{KP}{2}\frac{W}{L}\right) v_{id}^2$

11) $$I_T - \left(\frac{KP}{2}\frac{W}{L}\right) v_{id}^2 = 2\sqrt{i_1 i_0 + i_1^2}$$

square gives a quadratic in i_1

12) $$i_1^2 + i_0 i_1 - \frac{1}{4}\left(I_T - \left[\frac{KP}{2}\frac{W}{L}\right) v_{id}^2\right)^2 = 0$$

13) $$i_1 = -\frac{i_0}{2} \pm \frac{1}{2}\sqrt{i_0^2 + \left\{\left(I_T - \left(\frac{KP}{2}\frac{W}{L}\right)\right) v_{id}^2\right\}^2}$$

14) $$i_2 = i_0 + i_1 = +\frac{i_0}{2} \pm \frac{1}{2}\sqrt{i_0^2 + \left(I_T - \left(\frac{KP}{2}\frac{W}{L}\right) v_{id}^2\right)^2}$$

$\therefore$ $i_0 = i_2 - i_1 =$ checks above $\Rightarrow$ to $I_T = i_1 + i_2$

$\Rightarrow$ 15) $$I_T = \pm\frac{1}{2} \times 2\sqrt{i_0^2 + \left(I_T - \left(\frac{KP}{2}\frac{W}{L}\right) v_{id}^2\right)^2}$$

square again

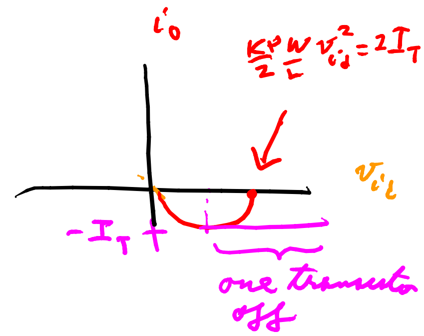
16) $$\cancel{I_T^2} = i_0^2 + (\cancel{I_T^2} - 2\left(\frac{KP}{2}\frac{W}{L}\right) v_{id}^2 I_T + \left(\frac{KP}{2}\frac{W}{L}\right)^2 v_{id}^4$$

17) $$i_0 = \pm\sqrt{-\left(\frac{KP}{2}\frac{W}{L} v_{id}^2\right)\left(\left(\frac{KP}{2}\frac{W}{L} v_{id}^2\right) - 2I_T\right)}$$

$$= -v_{id}\sqrt{\left(\frac{KP}{2} W/L\right)\left(2I_T - \left(\frac{KP}{2}\frac{W}{L}\right) v_{id}^2\right)}$$

find v_{id} to give $i_0 = \pm I_T$

$$I_T^2 = -x^2(x^2 - 2I_T) \text{ where } x^2 = \frac{KP}{2}\frac{W}{L} v_{id}^2$$

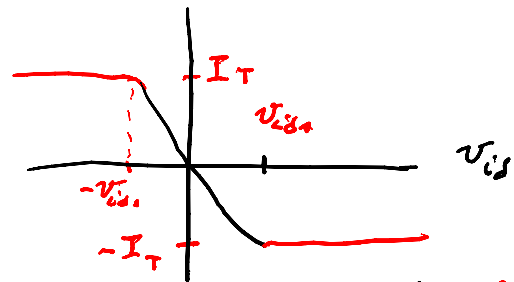


$$x^4 - 2I_T x^2 + I_T^2 = 0 \Rightarrow x^2 = I_T \pm \frac{1}{2}\sqrt{4I_T^2 - 4I_T^2}$$

$$= I_T \qquad \Rightarrow x^2 = I_T$$

(as x is real so rule out − sign)

$$\Rightarrow v_{id_S} = \pm\sqrt{\frac{2L}{WKP}}\sqrt{I_T}$$



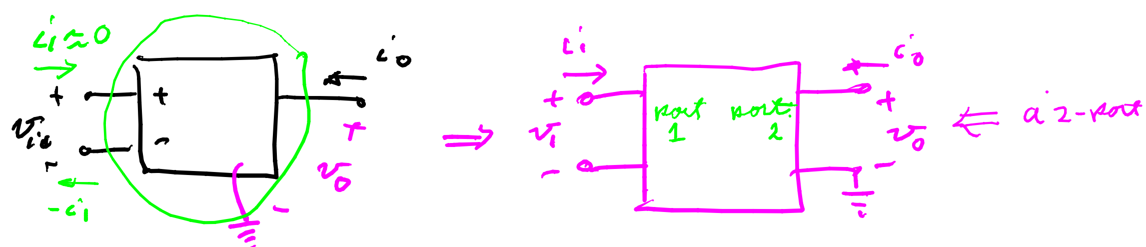
To find Gm: $$\frac{di_o}{dv_{id}} \overset{(7)}{=} \frac{d}{dx}\left(\pm\sqrt{x^2(x^2-2I_T)}\right)\cdot\frac{dx}{dv_{id}} \; ; \; x^2 = \frac{KP}{2}\frac{W}{L}v_{id}^2$$

$$= \pm\frac{1}{\sqrt{-x^2(x^2-2I_T)}}\cdot(-4x^3+4xI_T)\frac{1}{2}\cdot\sqrt{\frac{KP}{2}\frac{W}{L}}$$

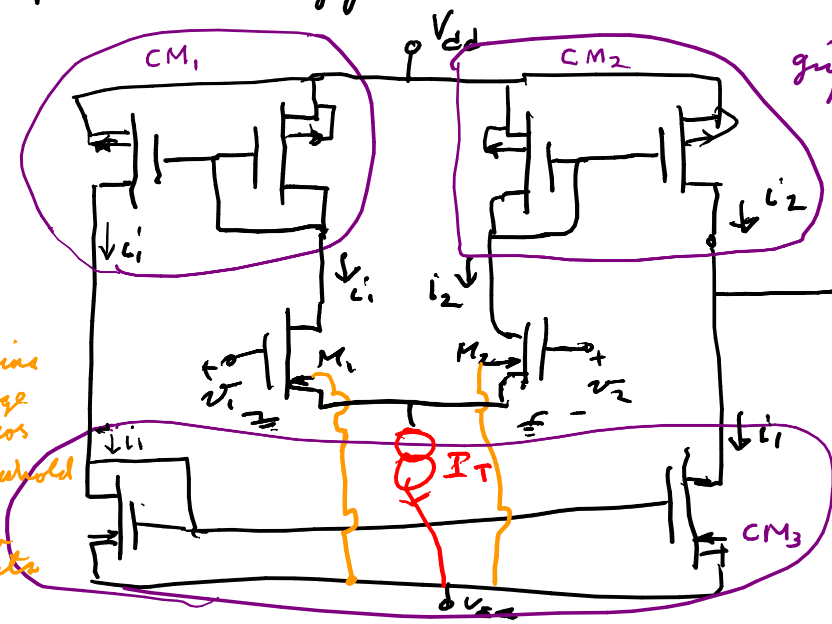
(note @ $x=0$ this is $0/0 \Rightarrow \neq \infty$ or 0)

$$= \pm\frac{-4x^2+4I_T}{\sqrt{-x^2+2I_T}}\cdot\sqrt{\frac{KP}{2}\frac{W}{L}} = -4\sqrt{\frac{KP}{2}\frac{W}{L}}\left(\frac{I_T-\left(\frac{KP}{2}\frac{W}{L}\right)v_{id}^2}{\sqrt{2I_T-\left(\frac{KP}{2}\frac{W}{L}\right)v_{id}^2}}\right) \underset{v_{id}=0}{\Rightarrow} \left|-2\sqrt{KP\left(\frac{W}{L}\right)I_T}\right|$$

not normally shown but have a current path to ground



Means around different loading of M_1 & M_2



gives the same load on M_1 & M_2 use 3 CMs

$i_o = i_1 - i_2$

substrate bias doesn't change OTA behavior as VTO → Vthreshold in M_1 & M_2 equally so still subtracts in Vid

makes a good OTA