EE 434
02/27/08

$$C\frac{du}{dt} = -Gu + Wa + I \quad , \quad a = f(u)$$

$$u_i = f_i^{-1}(a_i)$$

$$\mathcal{L}(u) = -a^T \frac{W}{2} a - a^T I + \sum_{i=1}^{N} G_i \int_0^{a_i} f_i^{-1}(x_i)\,dx_i$$ Lyapunov function

$\mathcal{L}(u)$ is bounded below

$$\frac{d\mathcal{L}(u)}{dt} = -\dot{a}^T\left(\frac{W}{2}a\right) - a^T \frac{W}{2}\dot{a} - \dot{a}^T I + \sum_{i=1}^{N} G_i \frac{d\int_0^{a_i} f^{-1}(x_i)\,dx_i}{dt} \quad ; \quad \dot{} = \frac{d}{dt}$$

$$= -\dot{a}^T\left(\frac{W}{2}a\right) - \dot{a}^T \frac{W^T}{2} a - \dot{a} I + \underbrace{\sum_{i=1}^{N} G_i \cdot \underbrace{\frac{d}{da_i}\int_0^{a_i} f_i^{-1}(x_i)\,dx_i}_{f_i^{-1}(a_i) = u_i} \cdot \frac{da_i}{dt}}_{\dot{a}^T \cdot G \cdot u}$$

G = diag matrix of G_i

C = diag matrix of C_i

$$\frac{d\mathcal{L}(u)}{dt} = -\dot{a}^T\left[\left(\frac{W}{2} + \frac{W^T}{2}\right)a + I - Gu\right]$$

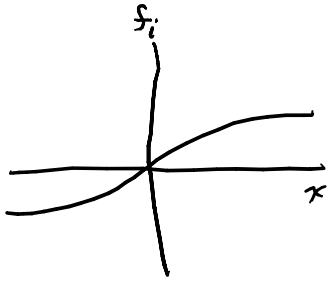
$$= -\dot{a}^T\left[W_{sym}\, a + I - Gu\right]$$

if $W = \frac{W + W^T}{2}$, i.e., W is symmetric

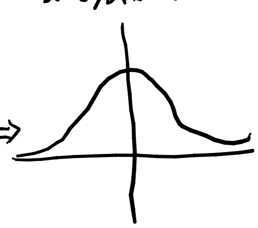
$$= -\dot{a}^T[\underbrace{Wa + I - Gu}_{C\,du/dt}] \quad ; \quad a = f(u)$$

$$\frac{da_i}{dt} = \frac{df_i}{du_i}\cdot\frac{du_i}{dt}$$

check $\frac{df_i}{du_i}$: for $f_i(x) = \tanh(x)$



$\Rightarrow$



$$\dot{a} = \begin{bmatrix} \frac{df_1}{du_1} & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \vdots & & & \\ 0 & & & \frac{df_N}{du_N} \end{bmatrix} \dot{u} = F \cdot \dot{u}$$

$$\dot{u} = F^{-1}\dot{a} \quad ; \quad F_{ii}^{-1} = \frac{1}{\frac{df_i}{du_i}} > 0$$

$$\frac{d\mathcal{L}(u)}{dt} = -\dot{a}^T C \dot{u} = -\dot{a}^T C F^{-1} \dot{a} = -\dot{a}^T D \dot{a} \quad ; \quad D_{ii} = C_i \Big/ \frac{df_i}{du_i} > 0$$

$D_{ij} = 0,\ i \neq j$

$\therefore$ $\dot{a}^T D \dot{a}$ is a positive definite quadratic form > 0 if $\dot{a} \neq 0$

$\therefore$ $\dfrac{d\mathcal{L}(u)}{dt} = -\dot{a}^T D \dot{a} < 0$ if $\dot{a} \neq 0$

$\Rightarrow$ implies the Hopfield neural network is stable & solutions go to equilibria.
if $W = W^T$, $f_i(x)$ are monotically increasing

We can always choose $W = W^T$

To make with hardware:

$$C\frac{du}{dt} + Gu = Wi + I \qquad i = f(u)$$

u = voltage on capacitors

Gu: Voltage controlled current source (linear)

Wi: current to current = current mirrors

I: DC current sources

$i = f(u)$: voltage controlled current sources (to give tanh or close to tanh) differential pairs; OTA

$\therefore$ can set up in Spice and with transistors (CMOS)