EE434
02/20/08

matlab call

num2cell(·)
cell2mat(·)

con2seq(·)　　con = concurrent
seq2con(·)　　seq = sequence

Hopfield

$$C_i \frac{du_i}{dt} + G_i u_i = I_i + \sum_{j=1}^{N} T_{ij} V_j \; , \quad V_j = \pm g(u_j) \; ; \text{ needs } u_i(0)$$

$i = 1, \ldots, N$

change notation: $V_j \Rightarrow i_j$, $T_{ij} \Rightarrow W_{ij}$

$$C \frac{du}{dt} + Gu = Wi + I \; ; \quad i = g(u) \; ;$$

$g(\cdot)$ = voltage controlled current (vector function)

$g(\cdot)$ = saturating functions ($\tanh(\cdot)$)

C = positive definite ($C_{ii} > 0$, $C_{ij} = 0$ if $j \neq i$)

G = positive semidefinite ($G_{ii} \geq 0$, $G_{ij} = 0$ if $j \neq i$)

C = capacitor matrix N×N, diagonal

u = vector of voltages on capacitors

G = diagonal matrix

W = weight matrix of currents to currents current mirrors

Need this to be stable; i.e. always go to finite rest points
‖
equilibria

$$\Rightarrow \text{when } \frac{du}{dt} = 0 \Rightarrow W i_{eq} = G u_{eq} - I$$

$$= W g(u_{eq}) = G u_{eq} - I$$

we specify u_{eq}: $W \Rightarrow W = [G u_{eq} - I] \, g^{-1}(u_{eq})$ can't do as $g(\cdot)$ is a vector

$$W [\, g(u_{eq_1}) \;\; g(u_{eq_2}) \,] = [\, G u_{eq_1} - I \, , \, G u_{eq_2} - I \,] \quad \text{for } N = 2$$

still won't work as I not known. If subtract can eliminate $I \Rightarrow$ need u_{eq_3} for $N=2$

$$W\left[g(u_{eq_1})-g(u_{eq_3}) \quad g(u_{eq_2})-g(u_{eq_3})\right] = \left[G(u_{eq_1}-u_{eq_3}) \quad G(u_{eq_2}-u_{eq_3})\right]$$

$$\therefore W = G\left[u_{eq_1}-u_{eq_3} \quad u_{eq_2}-u_{eq_3}\right]\left[g(u_{eq_1})-g(u_{eq_3}) \quad g(u_{eq_2})-g(u_{eq_3})\right]^{-1}$$

gives W so can get I

$$I = G u_{eq} - W g(u_{eq}) \qquad \text{for any } u_{eq}.$$

Ex: Choose $u_{eq_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $u_{eq_2} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $u_{eq_3} = \begin{bmatrix} -1/2 \\ +1/2 \end{bmatrix}$

$$u_{eq_1} - u_{eq_3} = \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}; \quad u_{eq_2} - u_{eq_3} = \begin{bmatrix} +1/2 \\ -3/2 \end{bmatrix}$$

$$\left[u_{eq_1} - u_{eq_3} \quad u_{eq_2} - u_{eq_3}\right] = \begin{bmatrix} 3/2 & +1/2 \\ -1/2 & -3/2 \end{bmatrix}; \ \det(\cdot) = -\frac{9}{4} + \frac{1}{4} = -2 \neq 0$$

$\therefore$ a nonsingular matrix

assume $G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

let $g_i(u_i) = \tanh(u_i)$; $\tanh(u_{eq_1}) = \begin{bmatrix} \tanh(1) \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ \tanh(-1) \end{bmatrix} = \begin{bmatrix} 0 \\ -\tanh(1) \end{bmatrix} = \tanh(u_{eq_2})$

$$\tanh(u_{eq_3}) = \begin{bmatrix} \tanh(-1/2) \\ \tanh(+1/2) \end{bmatrix} = \begin{bmatrix} -\tanh(1/2) \\ \tanh(1/2) \end{bmatrix}$$

$\tanh(1) = 0.7616$
$\tanh(1/2) = 0.4621$

$$\left[g(u_{eq_1}) - g(u_{eq_3})\right] = \begin{bmatrix} 0.7616 - (-0.4621) \\ 0 - 0.4621 \end{bmatrix} = \begin{bmatrix} 1.2237 \\ -0.4621 \end{bmatrix}$$

$$\left[g(u_{eq_2}) - g(u_{eq_3})\right] = \begin{bmatrix} 0 - (-0.4621) \\ -0.7616 - 0.4621 \end{bmatrix} = \begin{bmatrix} 0.4621 \\ -1.2237 \end{bmatrix}$$

$$\left[g(u_{eq_1}) - g(u_{eq_3}) \quad g(u_{eq_2}) - g(u_{eq_3})\right] = \begin{bmatrix} 1.2237 & 0.4621 \\ -0.4621 & -1.2237 \end{bmatrix}; \ \det = -(1.2237)^2 + (0.4621)^2$$

0.9531 0.3599
-0.3599 -0.9531

$$W = G\begin{bmatrix} 3/2 & 1/2 \\ -1/2 & -3/2 \end{bmatrix}\begin{bmatrix} 0.9531 & 0.3599 \\ -0.3599 & -0.9531 \end{bmatrix}$$

$$W = \begin{bmatrix} 1.2497 & 0.0633 \\ 0.0633 & 1.2497 \end{bmatrix}$$

1.2497 0.0633
0.0633 1.2497

$$I = G u_{eq_1} - W g(u_{eq_1}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1.2497 & 0.0633 \\ 0.0633 & 1.2497 \end{bmatrix}\begin{bmatrix} \tanh(1) \\ \tanh(0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 - (1.2497)(0.7616) - (0.0633)\times 0 \\ 0 - (0.0633)(0.7616) \end{bmatrix}$$

$$= \begin{bmatrix} 0.0482 \\ -0.0482 \end{bmatrix}$$

$$g(u) = \begin{bmatrix} \tanh(u_1) \\ \tanh(u_2) \end{bmatrix} \quad ; \quad \text{assume } C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{du}{dt} + u = \begin{bmatrix} 1.2497 & 0.0633 \\ 0.0633 & 1.2497 \end{bmatrix}\begin{bmatrix} \tanh u_1 \\ \tanh u_2 \end{bmatrix} + \begin{bmatrix} 0.0482 \\ -0.0482 \end{bmatrix}$$

02/27/08