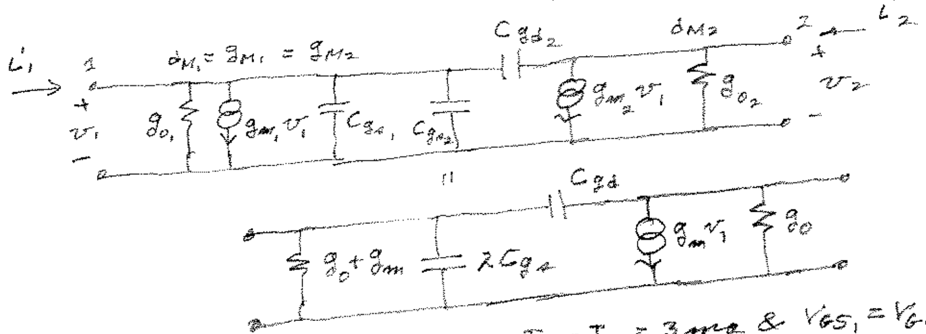
Midterm Solutions
EE303 Spring 2008

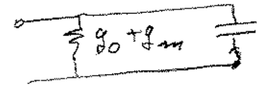
03/20/08
v.1

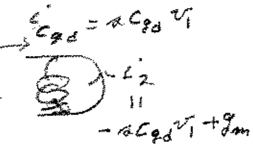
#1. a)



(Here $g_{o_1} = g_{o_2}$, $g_{m_1} = g_{m_2}$ as $I_1 = I_2 = 3$ ma & $V_{GS_1} = V_{GS_2}$)

b) $Y(s) \Rightarrow$ $y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0} \Rightarrow$

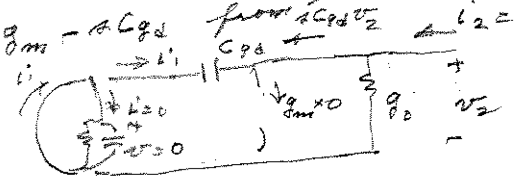




$= g_o + g_m + 3sC_{gs}$

$y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0} = g_m - sC_{gd}$ from $sC_{gd} v_2$ ← $i_2 = g_o v_2 + sC_{gd} v_2$

$y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0} \Rightarrow$



$= -sC_{gd}$ $\qquad y_{22} = g_o + sC_{gd}$

or
$$Y(s) = \begin{bmatrix} g_o + g_m + 3sC_{gs} & -sC_{gd} \\ g_m - sC_{gd} & g_o + sC_{gd} \end{bmatrix}$$

In terms of Spice parameters

$$g_m = \frac{\partial i_d}{\partial v_{gs}}\Big|_Q = 2\frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO})(1 + \lambda V_{GS}) = 2I_1/(V_{GS} - V_{TO}) \approx 2\frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO})$$

$$g_o = \frac{\partial i_d}{\partial v_{ds}}\Big|_Q = \lambda \cdot \frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO})^2 = \lambda I_1/(1 + \lambda V_{GS})$$

where also $I_1 \approx \frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO})^2 \Rightarrow V_{GS} \approx V_{TO} + \sqrt{I_1/(\frac{KP}{2}W/L)}$

one form of $Y(s)$:

$$Y(s) = \begin{bmatrix} \frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO})[2 + \lambda(V_{GS} - V_{TO})] + s(3C_{gs}) & -sC_{gs} \\ 2\frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO}) - sC_{gs} & \lambda\frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO})^2 + sC_{gs} \end{bmatrix}$$

c) Numerically: $V_{GS} \approx V_{TO} + \sqrt{I_1/(\frac{KP}{2}\frac{W}{L})} = 0.8 + \sqrt{\frac{3\times10^{-3}}{10^{-5}}} = 0.8 + \sqrt{3}\times10 =$
≈ 18.12 & $\lambda V_{DS} = \lambda V_{GS} = 0.036$ (so only small correction to V_{GS})

use $V_{GS} = 18.1$

$g_m = 2(10^{-5})(17.3) = 34.6\times10^{-5}$ ℧, $g_o = 2\times10^{-3} \cdot I_1 = 2\times3\times10^{-3}\times10^{-3} = 6\times10^{-6}$
$= 0.6\times10^{-5}$

$C_{gs} = C_{gd} = 5\times10^{-12}$

$$Y(s) = \begin{bmatrix} 35.2\times10^{-5} + s\times15\times10^{-12} & -s\times5\times10^{-12} \\ 34.6\times10^{-5} - s\times5\times10^{-12} & 6\times10^{-6} + s\times5\times10^{-12} \end{bmatrix}$$

poles @ $s = \infty$
zeros: (1,2) @ $s = 0$
(1,1) @ $s = -35/15\times10^{7}$
(2,1) @ $s = +35/5\times10^{7}$
(2,2) @ $s = -6/5\times10^{6}$

d) $v_2 = -g_L i_2 = (g_m - sC_{gd})v_1 + (g_o + sC_{gd})v_2 \Rightarrow \frac{v_2}{v_1} = -\frac{g_m - sC_{gd}}{g_o + g_L + sC_{gd}} = G(s)$

numerically $g_o + g_L = 2\times10^{-3} + 0.006\times10^{-3} \approx 2\times10^{-3}$

$$\therefore G(s) = \frac{s\times5\times10^{-12} - 34.6\times10^{-5}}{s\times5\times10^{-12} + 2\times10^{-3}} = \frac{s - 6.92\times10^{7}}{s + 0.4\times10^{9}}$$

@ low frequencies $G(0) \approx -\frac{g_m}{g_L} = -0.173$ (or a loss)

EE303 midterm solutions 03/20/08 V.2

#2. a) We need $-I_E = I_S \cdot e^{+V_{BE}/V_T} = 10^{-14} \times e^{+0.7/0.026} = 10^{-14} e^{+26.92}$
$= 10^{-14} \times 492.7 \times 10^{9} = 4.93 \times 10^{-3}$

$\therefore R_g \cdot (-I_E) = 2 = V_{GS} \Rightarrow R_g = \frac{2}{4.93 \times 10^{-3}} = 0.406 \times 10^{3} = 406\Omega = R_g$

For R_E; $V_{RE} = 10 - (2 + 0.7) = 7.3V$
$R_E \cdot (-I_E) = 7.3 \Rightarrow R_E = \frac{7.3}{4.93 \times 10^{-3}} = 1.58 \times 10^{3} = 1.58 K\Omega = R_E$

b) For M in saturation $V_{DS} \geq V_{GS} - V_{TO} = 2 - 0.8 = 1.2V$
When R_L increases V_{DS} decreases when in saturation, since I_D remains close to constant (as V_{GS} = constant = 2V).
Therefore R_{Lmax} occurs when $V_{DS} = V_{GS} - V_{TO} = 1.2V$
and then $V_{R_{Lmax}}$ (+ at top, − at bottom) $= 10 - 1.2 = 8.8V$.
Further $I_D = \frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO})^2(1 + \lambda[V_{GS} - V_{TO}])$
$= 10^{-5}(1.2)^2(1 + 2\times 10^{-3}[1.2]) = 1.44 \times 10^{-5} \times (1.0024)$
$\approx 14.4 \mu A$

$\therefore R_{Lmax} = \frac{8.8V}{14.4 \times 10^{-6}} = 0.611 \times 10^{6} = 611 \times 10^{3} = 611 K\Omega$

c) $V_{GS} - V_{TO}$ is fixed so
$I_D = 14.4\mu A\,(1 + \lambda V_{DS}) = 14.4 \times 10^{-6}(1 + 2\times 10^{-3} V_{DS})$
and $V_{DS} = (10 - R_L I_D) = 10 - R_L \times 14.4 \times 10^{-6}(1 + 2\times10^{-3}V_{DS})$
$\Rightarrow V_{DS}(1 + R_L \times 28.8 \times 10^{-9}) = 10 - 14.4 \times 10^{-6} R_L$
$\Rightarrow V_{DS} = \frac{10 - 14.4 \times 10^{-6} R_L}{1 + R_L \times 28.8 \times 10^{-9}}$ for $0 \leq R_L \leq R_{Lmax} = 611 \times 10^{3}$

or $i_D = 14.4 \times 10^{-6}\left(1 + 2\times 10^{-3}\left[\frac{10 - 14.4\times 10^{-6} R_L}{1 + R_L \times 28.8\times 10^{-9}}\right]\right)$
$= 14.4 \times 10^{-6}\left(\frac{(1 + 20\times 10^{-3}) + [28.8\times10^{-9} - 28.8\times 10^{-9}]R_L}{1 + 28.8\times 10^{-9} R_L}\right)$
$= \frac{14.7 \times 10^{-6}}{1 + 28.8 \times 10^{-9} R_L}$, $0 \leq R_L \leq 611 \times 10^{3}$

(as a check $14.7/(1 + 28.8 \times 0.611 \times 10^{-3}) = 14.4$)

Sketch