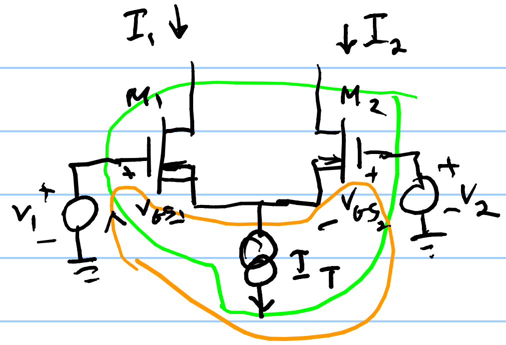
MOS differential pair: pp. 693-699

EE303
03/06/08



$-V_1 + V_{GS_1} - V_{GS_2} + V_2 = 0$ KVL

$V_{GS_1} - V_{GS_2} = V_1 - V_2 = V_{id}$

KCL (@ DC) $I_1 + I_2 - I_T = 0$

I_1, I_2 assume $M_1 = M_2$ in saturation & Early effect negligible

$I_1 = \frac{KP}{2}\frac{W}{L}(V_{GS_1} - V_{TH})^2$; $I_2 = \frac{KP}{2}\frac{W}{L}(V_{GS_2} - V_{TH})^2$; $V_{TH} = V_{TO}$ if $B = S$

$+\sqrt{\frac{I_1}{\frac{KP}{2}\frac{W}{L}}} = V_{GS_1} - V_{TH}$ $\qquad +\sqrt{\frac{I_2}{\frac{KP}{2}\frac{W}{L}}} = V_{GS_2} - V_{TH}$

$$V_{id} = V_{GS_1} - V_{GS_2} = \frac{1}{\sqrt{\frac{KP}{2}\frac{W}{L}}}\left(\sqrt{I_1} - \sqrt{I_2}\right)$$

$$\underbrace{I_1 + I_2}_{= I_T} - 2\sqrt{I_1 I_2} = \frac{KP}{2}\frac{W}{L}V_{id}^2 \Rightarrow \sqrt{I_1 I_2} = \frac{I_T}{2} - \frac{KP}{4}\frac{W}{L}V_{id}^2$$

$$I_1 I_2 = I_1(I_T - I_1) = -I_1^2 + I_T I_1 = \frac{1}{4}\left(I_T - \frac{KP}{2}\frac{W}{L}V_{id}^2\right)^2$$

$$I_1^2 - I_T I_1 + \frac{1}{4}\left(I_T - \frac{KP}{2}\frac{W}{L}V_{id}^2\right)^2$$

$$I_1 = \frac{I_T}{2} \pm \frac{1}{2}\sqrt{I_T^2 - \left(I_T^2 - 2I_T\frac{KP}{2}\frac{W}{L}V_{id}^2 + \left(\frac{KP}{2}\frac{W}{L}\right)^2 V_{id}^4\right)}$$

$$= \frac{I_T}{2} \pm \frac{1}{2}V_{id}\sqrt{2I_T\frac{KP}{2}\frac{W}{L} - \left(\frac{KP}{2}\frac{W}{L}\right)^2 V_{id}^2}$$

$$= \frac{I_T}{2} + \frac{1}{2}V_{id}\cdot\frac{KP}{2}\frac{W}{L}\sqrt{\frac{2I_T}{\frac{KP}{2}\frac{W}{L}} - V_{id}^2}$$

I_2 by symmetry: $I_2 = \frac{I_T}{2} - \frac{1}{2}V_{id}\left(\frac{KP}{2}\frac{W}{L}\right)\sqrt{\frac{2I_T}{\frac{KP}{2}\frac{W}{L}} - V_{id}^2}$

We know I_1 vs V_{id} is nondecreasing (& monotonic)

I_1 ceases to increase when $I_1 = I_T$

$\therefore$ look for $V_{id} > 0$ for which

$$\rightarrow \quad \frac{V_{id}}{2} \frac{KPW}{2L} \sqrt{2 I_T / \tfrac{KPW}{2L} - V_{id}^2} = \frac{I_T}{2}$$

$$V_{id}^2 \left(\frac{KPW}{2L}\right)^2 \cdot \left(\left(2 I_T / \tfrac{KPW}{2L}\right) - V_{id}^2 \right) = I_T^2 \qquad \text{let } x = V_{id}^2, \; b = \frac{KP}{2W/L}$$

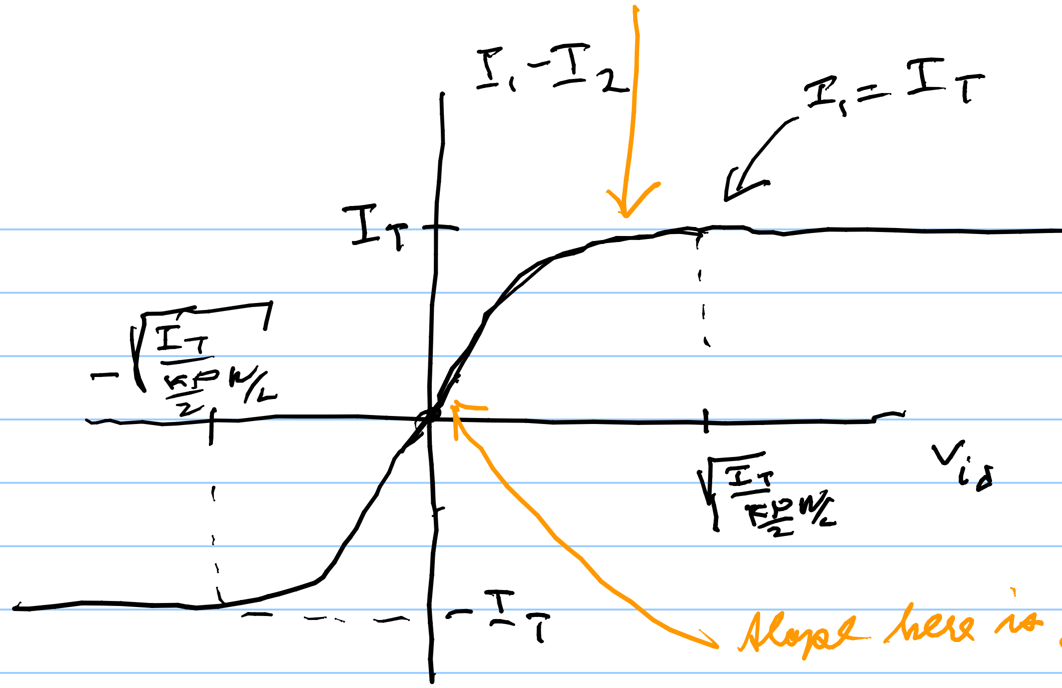
$$-x^2 \cdot b^2 + 2 I_T b \cdot x - I_T^2 = 0$$

$$x^2 - 2 I_T / b \cdot x + (I_T/b)^2 = 0$$

$$x = I_T / b \pm \frac{1}{2} \sqrt{4 \frac{I_T^2}{b^2} - 4 (I_T/b)^2} = \frac{I_T}{b} \pm \frac{I_T}{b} \sqrt{0}$$

$$V_{id}^2 = I_T / b = I_T / \tfrac{KPW}{2L} \; ; \quad V_{id} = \pm \sqrt{I_T / \tfrac{KPW/L}{2}}$$

$$I_1 - I_2 = V_{id} \left(\frac{KPW}{2L}\right) \sqrt{2 \frac{I_T}{\left(\frac{KPW}{2L}\right)} - V_{id}^2}$$

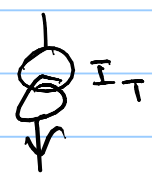


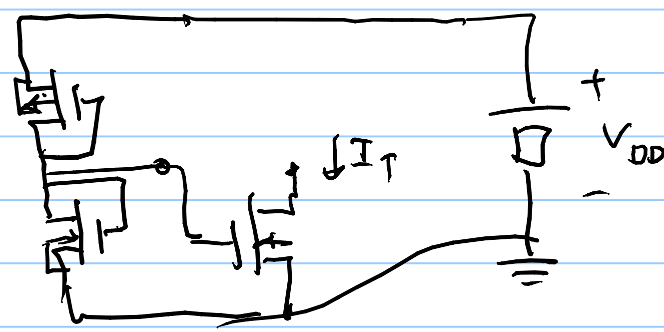
Slope here is $g_m = \frac{i_1 - i_2}{v_{id}}$

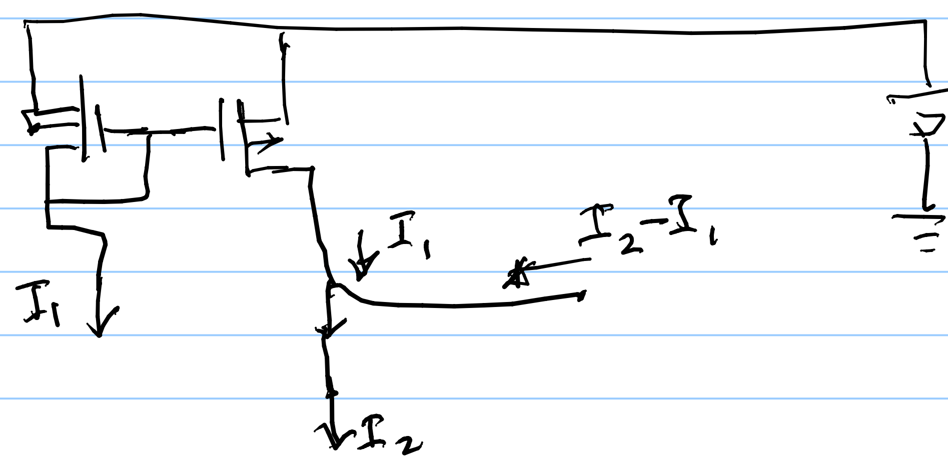
$$= \frac{KPW}{2L}\sqrt{\frac{2I_T}{\frac{KPW}{2L}}}$$

$$= \sqrt{2I_T \cdot \frac{KPW}{2L}}$$

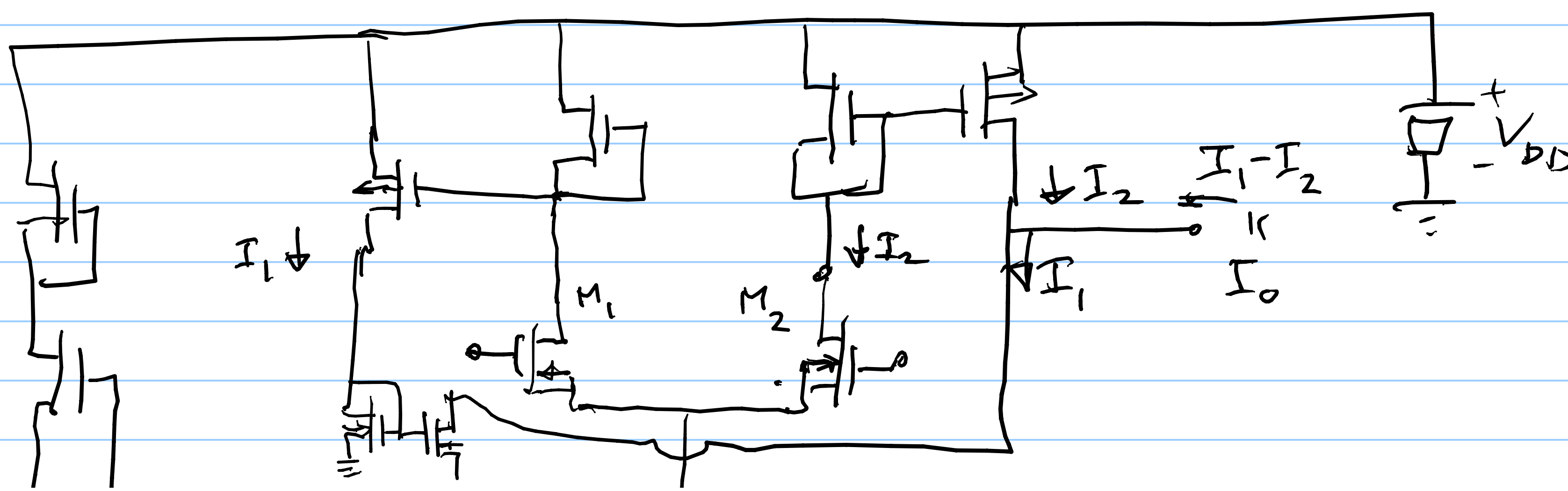
To create the tail current:







I1
I1
I2-I1
I2



I1
M1
M2
I2
I2
I1
I1-I2
I0
+
VDD
-