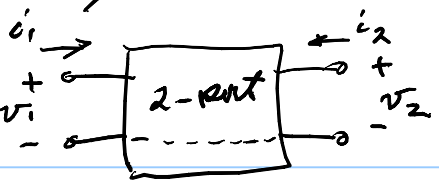
appendix B — 2-Port network parameters

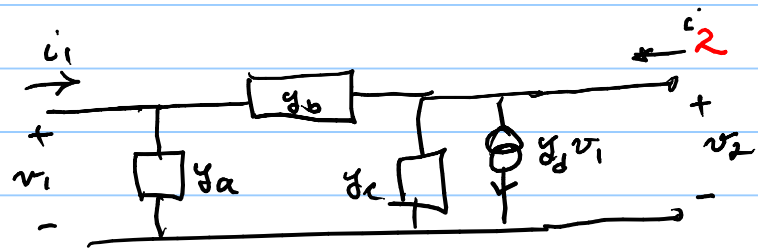
EE 303
07/19/08



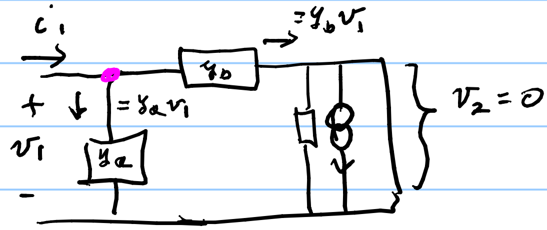
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \underbrace{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}}_{Y \text{ matrix}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

linear equations

Equivalent circuit



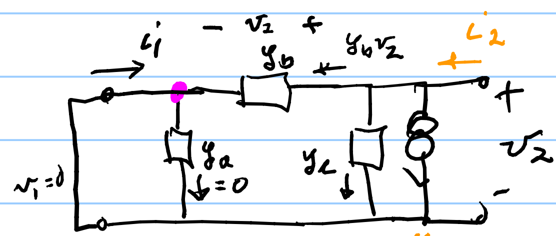
$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$$



$i_1 = y_a v_1 + y_b v_1$

$y_{11} = y_a + y_b$

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0}$$



"$i = 0 = y_d \cdot v_1 \big|_{v_1=0}$"

$i_1 = -y_b v_2$

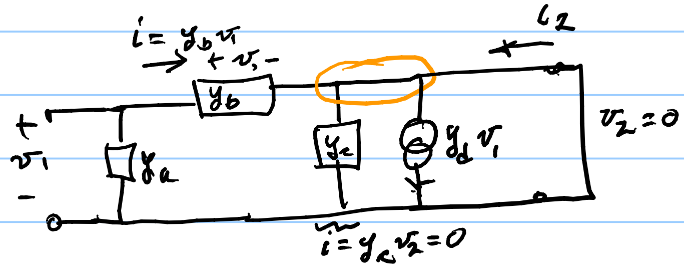
$y_{12} = -y_b$

$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

$i_2 = y_b v_2 + y_c v_2$

$y_{22} = y_b + y_c$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$



$i_2 = -y_b v_1 + y_d v_1$

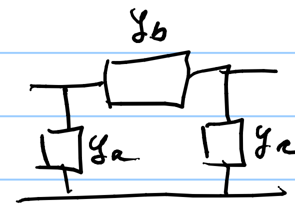
$y_{21} = -y_b + y_d$

$$Y = \begin{bmatrix} y_a + y_b & -y_b \\ -y_b + y_d & y_b + y_c \end{bmatrix} = \begin{bmatrix} y_b & -y_b \\ -y_b & y_b \end{bmatrix} + \begin{bmatrix} y_a & 0 \\ y_d & y_c \end{bmatrix}$$

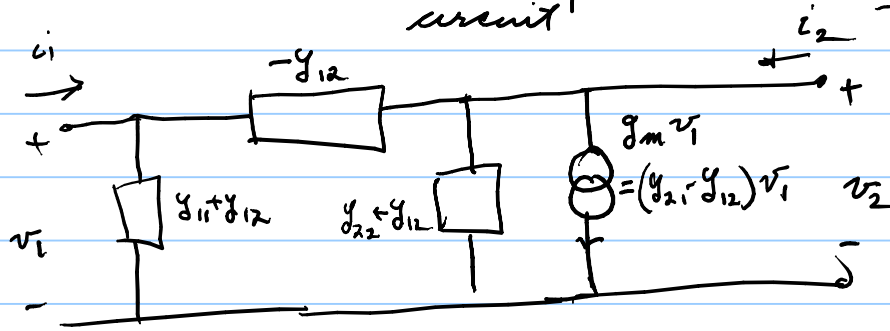
here $y_{12} = -y_b$ $\quad y_{11} = y_a + y_b \Rightarrow y_a = y_{11} + y_{12}$

$\Rightarrow y_b = -y_{12}$ $\quad y_{22} = y_c + y_b \Rightarrow y_c = y_{22} + y_{12}$

$y_{21} = y_d - y_b \Rightarrow y_d = y_{21} + y_b = y_{21} - y_{12}$

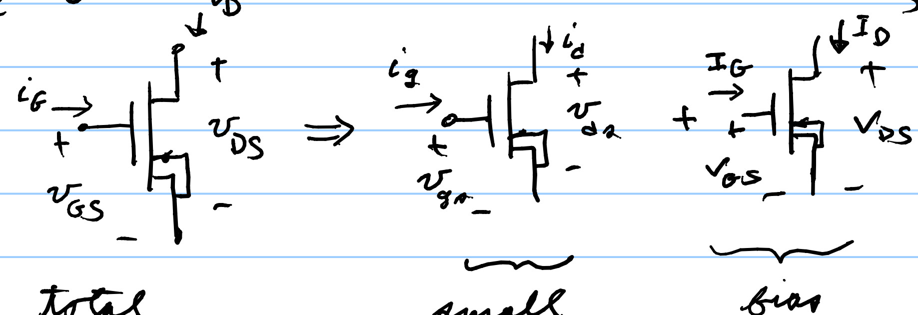


If $Y = Y^T$ (ie $y_{12} = y_{21}$) then $y_d = 0$ & we get the π equivalent circuit



total → small signal → bias

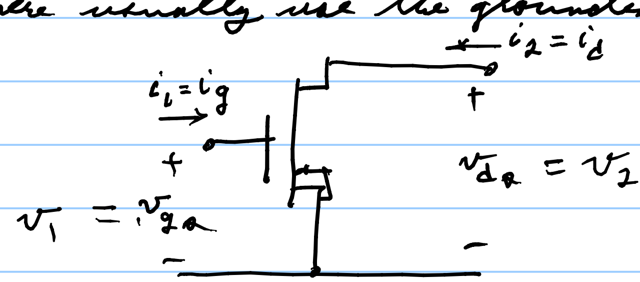
$i_G = i_g + I_G$ $\quad i_D = i_d + I_D$ $\quad v_{GS} = v_{gs} + V_{GS}$, $\quad v_{DS} = v_{ds} + V_{DS}$



$\Downarrow$

uses linear term in Taylor series expansion & use Y matrix to describe

here usually use the grounded source Y matrix



at low freq. $i_g = 0$ $\quad Y = \begin{bmatrix} 0 & 0 \\ & \end{bmatrix}$

If in saturation at Q point then $i_D = \frac{KP}{2}\frac{W}{L}(v_{GS} - V_{TO})^2(1 + \lambda v_{DS})$

$$i_D = I_D + \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q (v_{GS} - V_{GS}) + \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q (v_{DS} - V_{DS}) \quad + \text{(terms of higher order ignore for now)}$$

$$i_2 = i_d = i_D - I_D = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q v_{ga} + \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q v_{da} \Rightarrow i_2 = \underbrace{\left(\frac{\partial i_D}{\partial v_{GS}}\right)_Q}_{= y_{21}} v_1 + \underbrace{\left(\frac{\partial i_D}{\partial v_{DS}}\right)_Q}_{= y_{22}} v_2$$

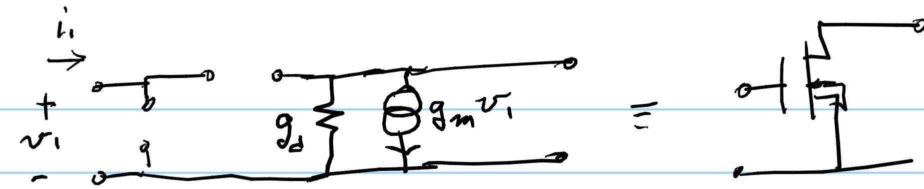
for our transistor in saturation

$$\left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q = \left.\frac{KP}{2}\frac{W}{L}\left[2(v_{GS} - V_{TO})(1 + \lambda v_{DS})\right]\right|_Q = y_{21} = KP \cdot \frac{W}{L}(V_{GS} - V_{TO})(1 + \lambda V_{DS})$$

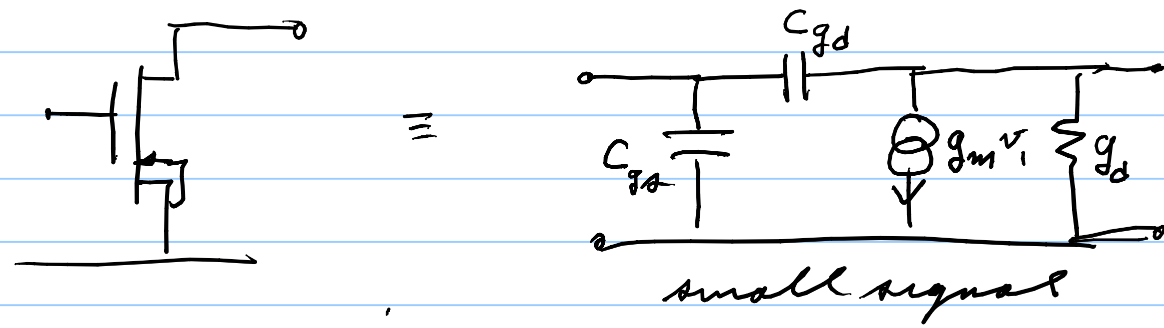
$$= g_m$$

$$\left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q = \left.\frac{KP}{2}\frac{W}{L}(v_{GS} - V_{TO})^2 \cdot \lambda\right|_Q = I_D \cdot \frac{\lambda}{1 + \lambda V_{DS}} = y_{22} = g_d$$

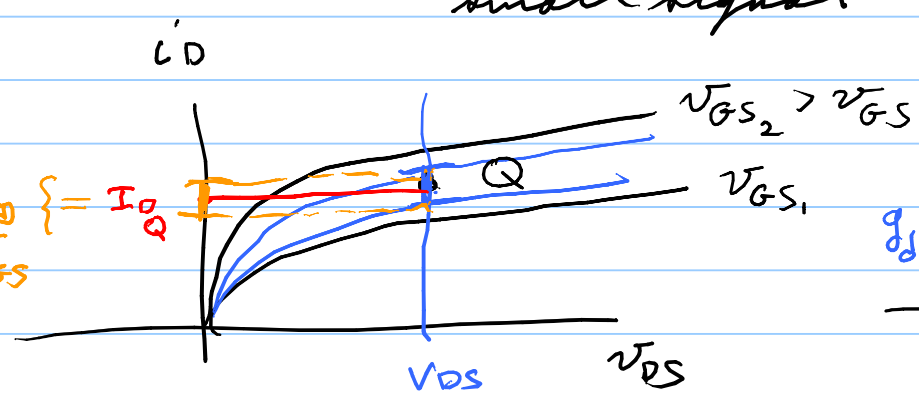
$$Y = \begin{bmatrix} 0 & 0 \\ g_m & g_d \end{bmatrix} \Rightarrow$$



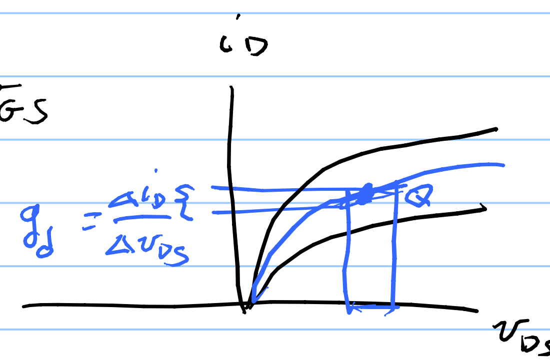
at higher frequency we see gate capacitance; add C_{gd}, C_{gs}



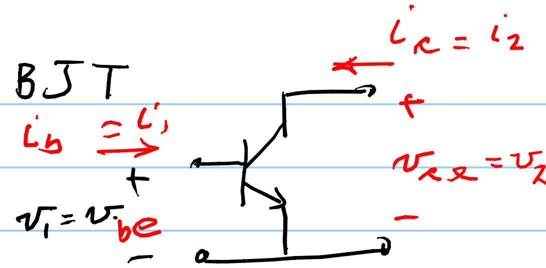
$$g_m = \frac{\Delta i_D}{\Delta v_{GS}} \Big\} = I_{D_Q}$$



$$g_d = \frac{\Delta i_D}{\Delta v_{DS}} \Big\}$$



For BJT



$\overset{?}{=}$ Y matrix