File: e:/courses/spring2007/303/cubicsolv.doc 02/25/07 RWN

file:e:/courses/spring2007/303/cubicsolv.mcd RWN 02/25/07
Mathcad page for solving the cubic equation needed to find VSS for vin=vout=0, VDD given

$KPn := 4\cdot 10^{-4}$ $KPp := 2\cdot 10^{-4}$

$Wn := 100\cdot 10^{-6}$ $Ln := 10\cdot 10^{-6}$ $Wp := 400\cdot 10^{-6}$ $Lp := 10\cdot 10^{-6}$

$VTOn := 1.2$ $VAn := 100$ $VTOp := -1.5$ $VAp := 50$

$\lambda n := \frac{1}{VAn}$ $\lambda p := \frac{1}{VAp}$

$VDD := 5$

$kn := \left(\frac{KPn}{2}\right)\cdot\left(\frac{Wn}{Ln}\right)$ $kn = 2\cdot 10^{-3}$ $kp := \left(\frac{KPp}{2}\right)\cdot\left(\frac{Wp}{Lp}\right)$ $kp = 4\cdot 10^{-3}$

Set up equations to solve for x=-Vss to give vout=vo=vi=vin when vin=0
-IDp=kp(VDD-vi-|VTOp|)^2(1+λp[VDD-vo])=IDn=kn(vi+x-VTOn)^2(1+λn[vo+x])
which gives the cubic
x^3+(VAn-2VTOn)x^2+(VTOn^2-2VTOnVAn)x+VAn(VTOn^2-[-IDp/kn])
=x^3+a2x^2+a1x+a0

$vi := 0$ $vo := 0$

$a2 := VAn - 2\cdot VTOn$ $a2 = 97.6$

$a1 := VTOn^2 - 2\cdot VTOn\cdot VAn$ $a1 = -238.56$

$a0 := VAn\cdot\left[VTOn^2 - \left(\frac{kp}{kn}\right)\cdot(VDD - |VTOp|)^2\cdot(1+\lambda p\cdot VDD)\right]$

$a0 = -2.551\cdot 10^3$

$f(x) := x^3 + a2\cdot x^2 + a1\cdot x + a0$

Initial choice for x is found by solving with λn=0 so that the equation is quadratic
xo=VTOn+(-IDp/kn)^(1/2)

$xo := VTOn + \sqrt{\left[\left(\frac{kp}{kn}\right)\cdot(VDD - |VTOp|)^2\cdot(1+\lambda p\cdot VDD)\right]}$

$x := xo$ $xo = 6.391$

$root(f(x) - 0, x) = 6.237$

Therefore VSS=-6.237V give vout=vin=(

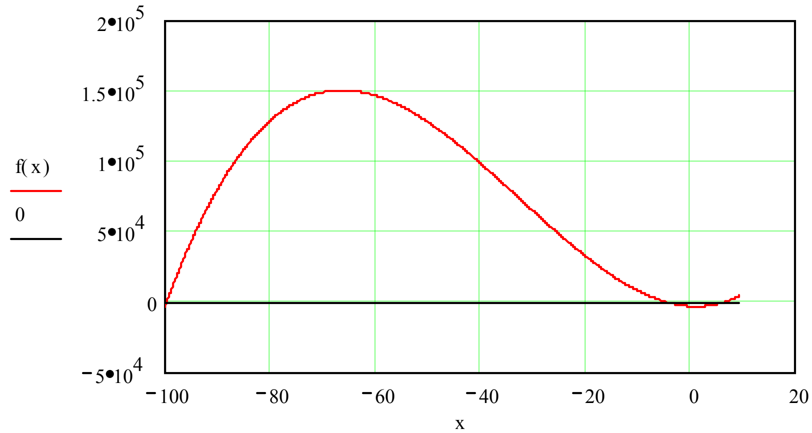
An alternate is to use the polynomial root finder polyroots(v) where v is the vector o polynomial coefficients, starting with the constant term

$v := \begin{bmatrix} a0 \\ a1 \\ a2 \\ 1 \end{bmatrix}$ $xv := polyroots(v)$ $xv = \begin{bmatrix} -99.735 \\ -4.101 \\ 6.237 \end{bmatrix}$

$xv_2 = 6.237$

$xmin := -100 \quad xinc := 0.1 \quad xmax := 3\cdot\frac{xo}{2}$

$x := xmin, xmin + xinc .. xmax$



$x1 := xv_2 - xinc \qquad x2 := xv_2 + xinc$