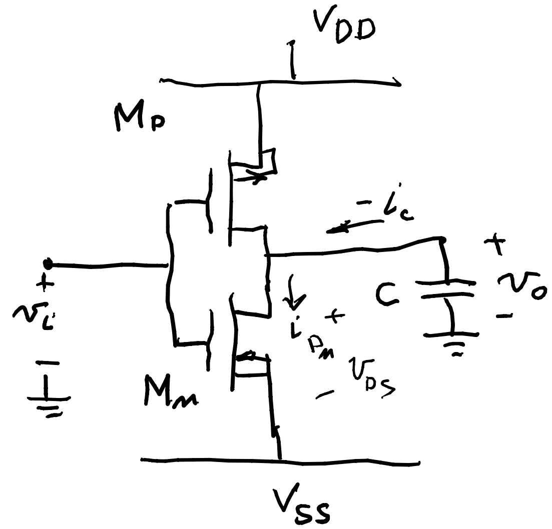
EE303
03/09/07



$v_o(0) = V_{DD}$

$v_i(t) = V_{SS}$ for $t<0$

$v_i(t) = V_{DD}$ for $t>0$

for $t>0$ M_p is off, $i_{D_p} \equiv 0$

at $t=0_+$ $\quad V_{GS_n} = v_i - V_{SS} = V_{DD} - V_{SS}$, $V_{DS_n} = v_o - V_{SS} = V_{DD} - V_{SS}$

$\therefore V_{GS_n} - VTO_n < V_{DS_n}$ if $VTO_n > 0$ (assume) enhancement mode

at $t=0_+$ M_n is in saturation

if ignore Early effect (i.e. set $\lambda_n = 0$)

$$i_{D_n} = \frac{KP}{2}\frac{W}{L}(v_{GS} - VTO)^2 = \frac{KP}{2}\frac{W}{L}(V_{DD} - V_{SS} - VTO)^2 = I_D$$

as long as $v_{GS} - VTO \le v_{DS}$

$$i_{D_n} = -i_C = -C\frac{dv_o}{dt}$$

$$\Rightarrow \frac{dv_o}{dt} = -\frac{I_D}{C}$$

$$v_o(t) = -\frac{I_D}{C}t + V_{DD}$$

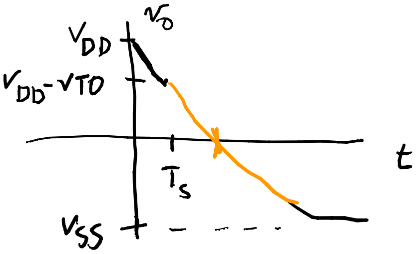
find the time, T_S, at which M_n goes out of saturation into ohmic region (triode, active)

$$v_{DS} = v_o - V_{SS} = v_{GS} - VTO$$
$$= V_{DD} - V_{SS} - VTO$$

$$v_o(T_S) = V_{DD} - VTO = -\frac{I_D}{C}T_S + V_{DD} \Rightarrow T_S = \frac{C \cdot VTO}{I_D}$$

for $t \geqslant T_s$

$$-i_D = c\frac{dv_o}{dt}$$



but are now in ohmic region for M_n $\quad v_{GS} - VTO \geqslant v_{DS} = v_o - V_{SS}$

$\overset{\shortparallel}{V_{DD}-V_{SS}}$

here $\quad i_D = \frac{KP}{2}\frac{W}{L}\left\{2(v_{GS}-VTO)v_{DS} - v_{DS}^2\right\}$

$$= \frac{KP}{2}\frac{W}{L}\left\{2(V_{DD}-V_{SS}-VTO)(v_o-V_{SS}) - (v_o-V_{SS})^2\right\}$$

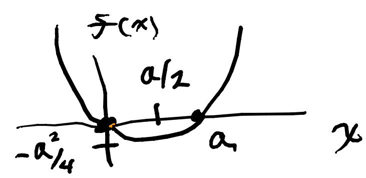
$\therefore$

$$\frac{dv_o}{dt} = \frac{d(v_o-V_{SS})}{dt} = \frac{-1}{C}\frac{KP}{2}\frac{W}{L}\left\{2(V_{DD}-V_{SS}-VTO)(v_o-V_{SS}) - (v_o-V_{SS})^2\right\}$$

$$\frac{dx}{dt} = -K\left\{a x - x^2\right\}, \quad x = v_o - V_{SS}$$
$$a = 2(V_{DD}-V_{SS}-VTO)$$
$$K = \frac{KP\,W}{2LC}$$

let $\hat{t} = Kt$ $\qquad$ at $t=T_s$, $x = v_o - V_{SS}\big|_{t=T_s} = V_{DD}-V_{SS}-VTO$

$$\frac{dx}{K\,dt} = \frac{dx}{d(Kt)} = \frac{dx}{d\hat{t}} = \mathring{x} = x(x-a) = f(x)$$



$$\frac{dx}{d\hat{t}} = x(x-a) \Rightarrow d\hat{t} = \frac{dx}{x(x-a)}$$
$$= \frac{\alpha}{x}dx + \frac{\beta\,dx}{x-a}$$

$$\int_{\hat{T}_s}^{\hat{t}} d\hat{t}_1 = \int_{x(\hat{T}_s)}^{x(\hat{t})} \frac{\alpha}{x_1}dx_1 + \int_{x(\hat{T}_s)-a}^{x(\hat{t})-a} \frac{\beta}{x_2-a}\,d(x_2-a) = \int_{x(\hat{T}_s)-a}^{x(\hat{t})-a} \frac{\beta}{x_3}dx_3$$

to get α & β $\quad \frac{1}{x(x-a)} = \frac{\alpha}{x} + \frac{\beta}{x-a}$

$$1 = (x-a)\alpha + x\beta \qquad @\, x=0 \quad 1 = -a\alpha$$
$$\alpha = -1/a$$
$$@\, x = a \quad 1 = a \cdot \beta \Rightarrow \beta = 1/a$$

$$\hat{t} - \hat{T}_s = -\frac{1}{a}\left(\ln\left(x_1\Big|^{x(\hat{t})}\right) - \ln\left(x\Big|^{x(\hat{T}_s)}\right)\right) + \frac{1}{a}\left(\ln\left(x_3\Big|^{x(\hat{t})-a}\right) - \ln\left(x_3\Big|^{x(\hat{T}_s)-a}\right)\right)$$

$$= \frac{1}{a}\ln\left(\frac{(x(\hat{t})-a)/(x(\hat{T}_s)-a)}{x(\hat{t})/x(\hat{T}_s)}\right) = \frac{1}{a}\ln\left(\frac{1-\frac{a}{x(\hat{t})}}{1-\frac{a}{x(\hat{T}_s)}}\right)$$

then take exponentials on both sides and solve for $x(\hat{t})$

$$e^{a(\hat{t}-\hat{T}_s)} \times \left(1 - \frac{a}{x(\hat{T}_s)}\right) = 1 - \frac{a}{x(\hat{t})} \Rightarrow \frac{1}{x(\hat{t})} = \frac{1}{a}\left[1 - \left(1 - \frac{a}{x(T_s)}\right)e^{a(\hat{t}-\hat{T}_s)}\right]$$

$$\Rightarrow \quad x(\hat{t}) = \frac{a}{1 - \left(1 - \frac{a}{x(\hat{T}_s)}\right)e^{a(\hat{t}-\hat{T}_s)}}$$

note as $\hat{t} \to \infty$, $x \to 0$
$\Rightarrow v_o \to v_{ss}$

and @ $\hat{t} \to \hat{T}_s$ $\quad x \to \dfrac{a}{1 - 1 + \frac{a}{x(\hat{T}_s)}} = x(\hat{T}_s)$