Program to calculate symmetric weights for storage of 4-vectors; array origin set to 1

We wish equilibrium points of Cdv/dt = Wy-Gv+I, y=y(v)

$C1 := 5 \quad C2 := 5 \quad C3 := 5 \quad C4 := 5$

$G1 := 2 \quad G2 := 2 \quad G3 := 2 \quad G4 := 2$

$$C := \begin{bmatrix} C1 & 0 & 0 & 0 \\ 0 & C2 & 0 & 0 \\ 0 & 0 & C3 & 0 \\ 0 & 0 & 0 & C4 \end{bmatrix} \qquad C = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$G := \begin{bmatrix} G1 & 0 & 0 & 0 \\ 0 & G2 & 0 & 0 \\ 0 & 0 & G3 & 0 \\ 0 & 0 & 0 & G4 \end{bmatrix} \qquad G = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Initial choice of equilibrium points, to be later augmented to obtain symmetry

$$v1 := \begin{bmatrix} 0.5 \\ 0.25 \\ 0 \\ 0 \end{bmatrix} \qquad v2 := \begin{bmatrix} -0.5 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} \qquad v3 := \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0.25 \end{bmatrix} \qquad v4 := \begin{bmatrix} 0 \\ 0 \\ -0.5 \\ 0.5 \end{bmatrix}$$

$$y(v) := \begin{bmatrix} \tanh(v_1) \\ \tanh(v_2) \\ \tanh(v_3) \\ \tanh(v_4) \end{bmatrix}$$

$y1 := y(v1) \qquad y2 := y(v2) \qquad y3 := y(v3) \qquad y4 := y(v4)$

$$y1 = \begin{bmatrix} 0.462117 \\ 0.244919 \\ 0 \\ 0 \end{bmatrix} \qquad y2 = \begin{bmatrix} -0.462117 \\ 0.462117 \\ 0 \\ 0 \end{bmatrix} \qquad y3 = \begin{bmatrix} 0 \\ 0 \\ 0.462117 \\ 0.244919 \end{bmatrix} \qquad y4 = \begin{bmatrix} 0 \\ 0 \\ -0.462117 \\ 0.462117 \end{bmatrix}$$

Now need to let vx be variable and choose it to give a symmetric W
Define an unknown as vx

$$vx(x1, x2, x3, x4) := \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} \qquad yx(x1, x2, x3, x4) := \begin{bmatrix} \tanh(x1) \\ \tanh(x2) \\ \tanh(x3) \\ \tanh(x4) \end{bmatrix}$$

Inset these vx unknowns into the v vectors so that the 0 assigned entries above will subtract to zero when form vi-vx:

$$vx1(x3,x4) := v1 + \begin{bmatrix} 0 \\ 0 \\ x3 \\ x4 \end{bmatrix} \quad vx2(x3,x4) := v2 + \begin{bmatrix} 0 \\ 0 \\ x3 \\ x4 \end{bmatrix}$$

$$vx3(x1,x2) := v3 + \begin{bmatrix} x1 \\ x2 \\ 0 \\ 0 \end{bmatrix} \quad vx4(x1,x2) := v4 + \begin{bmatrix} x1 \\ x2 \\ 0 \\ 0 \end{bmatrix}$$

There will be two sets of 2x2 matrices to set symmetric with one set being in terms of x1,x2 and the other in terms of x3,x4

Both sets of equations to solve come from

W(y1-yx,y2-yx,y3-yx,y4-yx)=G(v1-vx,v2-vx,v3-vx,v4-vx)

but this separates into two 2x2 matrices, so separate vx and yx into 2 parts

$$vxa(x1,x2) := \begin{bmatrix} x1 \\ x2 \end{bmatrix} \quad vxb(x3,x4) := \begin{bmatrix} x3 \\ x4 \end{bmatrix}$$

$$yxa(x1,x2) := \begin{bmatrix} \tanh(x1) \\ \tanh(x2) \end{bmatrix} \quad yxb(x3,x4) := \begin{bmatrix} \tanh(x3) \\ \tanh(x4) \end{bmatrix}$$

$$V1(x1,x2) := \begin{bmatrix} v1_1 - vxa(x1,x2)_1 & v2_1 - vxa(x1,x2)_1 \\ v1_2 - vxa(x1,x2)_2 & v2_2 - vxa(x1,x2)_2 \end{bmatrix}$$

$$Y1(x1,x2) := \begin{bmatrix} y1_1 - yxa(x1,x2)_1 & y2_1 - yxa(x1,x2)_1 \\ y1_2 - yxa(x1,x2)_2 & y2_2 - yxa(x1,x2)_2 \end{bmatrix}$$

setup for solving for x1 given x2 to make the upper 2x2 submatrix of W symmetric
We have W=GVY^-1 so form this product and set = its transpose (ignoring the common denominator (= determinant Y1)

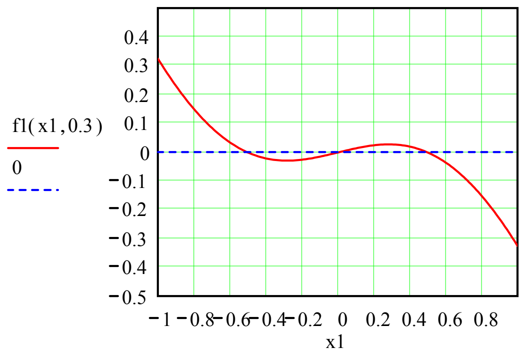
$$W12(x1) := G1\cdot(v2_1 - x1)\cdot(y1_1 - \tanh(x1)) - G1\cdot(v1_1 - x1)\cdot(y2_1 - \tanh(x1))$$

$$W21(x2) := G2\cdot(v1_2 - x2)\cdot(y2_2 - \tanh(x2)) - G2\cdot(v2_2 - x2)\cdot(y1_2 - \tanh(x2))$$

$$f1(x1,x2) := W12(x1) - W21(x2)$$

$x1min := -1 \quad x1max := 1 \quad x1inc := 0.01$

$x1 := x1min, x1min + x1inc .. x1max$



Solve block to determine x1 given x2; arbitrarily choose x2 as 0.3

Guess value: $x2 := 0.3$

$x1 := 0.5$

Given:

$x10 := \text{root}(f1(x1, 0.3), x1)$

$x10 = 0.494563$

$x1 := x10$

Form the upper 2x2 weight matrix which should be symmetric $G11 := \begin{bmatrix} G1 & 0 \\ 0 & G2 \end{bmatrix}$

$W1(x1, x2) := G11 \cdot V1(x1, x2) \cdot \left(Y1(x1, x2)^{-1}\right)$

$V1(x1, x2) = \begin{bmatrix} 5.436955 \cdot 10^{-3} & -0.994563 \\ -0.05 & 0.2 \end{bmatrix} \quad Y1(x1, x2) = \begin{bmatrix} 4.286609 \cdot 10^{-3} & -0.919948 \\ -0.046394 & 0.170805 \end{bmatrix}$

$W1(x1, x2) = \begin{bmatrix} 2.15568 & -0.035206 \\ -0.035213 & 2.1522 \end{bmatrix}$

Find the input I that goes with these by I=-Wy+Gv

$W1 := W1(x1, x2)$

$v1x := \begin{bmatrix} x1 \\ x2 \end{bmatrix} \quad v1x = \begin{bmatrix} 0.494563 \\ 0.3 \end{bmatrix}$

$y1x := \begin{bmatrix} \tanh(x1) \\ \tanh(x2) \end{bmatrix} \quad y1x = \begin{bmatrix} 0.457831 \\ 0.291313 \end{bmatrix}$

$I1 := -W1 \cdot y1x + G11 \cdot v1x \qquad I1 = \begin{bmatrix} 0.012446 \\ -0.010841 \end{bmatrix}$

Next do the same for the lower 2x2 submatrix, now solving for x3,x4

$$V2(x3,x4) := \begin{bmatrix} v3_3 - vxb(x3,x4)_1 & v4_3 - vxb(x3,x4)_1 \\ v3_4 - vxb(x3,x4)_2 & v4_4 - vxb(x3,x4)_2 \end{bmatrix}$$

$$Y2(x3,x4) := \begin{bmatrix} y3_3 - yxb(x3,x4)_1 & y4_3 - yxb(x3,x4)_1 \\ y3_4 - yxb(x3,x4)_2 & y4_4 - yxb(x3,x4)_2 \end{bmatrix}$$

setup for solving for x3 given x4 to make the lower 2x2 submatrix of W symmetric

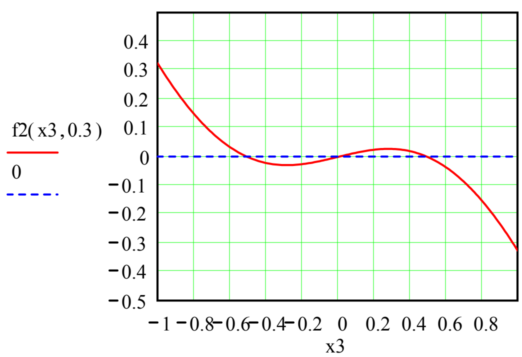
$$W34(x3) := G3 \cdot (v4_3 - x3) \cdot (y3_3 - \tanh(x3)) - G3 \cdot (v3_3 - x3) \cdot (y4_3 - \tanh(x3))$$

$$W43(x4) := G4 \cdot (v3_4 - x4) \cdot (y4_4 - \tanh(x4)) - G4 \cdot (v4_4 - x4) \cdot (y3_4 - \tanh(x4))$$

$$f2(x3,x4) := W34(x3) - W43(x4)$$

$x3min := -1$ $x3max := 1$ $x3inc := 0.01$

$x3 := x3min, x3min + x3inc .. x3max$



Solve block to determine x3 given x4

Guess value: $x4 := 0.3$

$x3 := 0.5$

Given:

$x30 := root(f2(x3, 0.3), x3)$

$x30 = 0.494563$ $x3 := x30$

Form the lower 2x2 weight matrix which should be symmetric $G22 := \begin{bmatrix} G3 & 0 \\ 0 & G4 \end{bmatrix}$

$$W2(x3,x4) := G22 \cdot V2(x3,x4) \cdot \left(Y2(x3,x4)^{-1}\right)$$

$$V2(x3,x4) = \begin{bmatrix} 5.436955 \cdot 10^{-3} & -0.994563 \\ -0.05 & 0.2 \end{bmatrix} \quad Y2(x3,x4) = \begin{bmatrix} 4.286609 \cdot 10^{-3} & -0.919948 \\ -0.046394 & 0.170805 \end{bmatrix}$$

$$W2(x3,x4) = \begin{bmatrix} 2.15568 & -0.035206 \\ -0.035213 & 2.1522 \end{bmatrix}$$

$$W2 := W2(x3, x4)$$

Find the input I that goes with these by I=-Wy+Gv

$$v3x := \begin{bmatrix} x3 \\ x4 \end{bmatrix} \quad v3x = \begin{bmatrix} 0.494563 \\ 0.3 \end{bmatrix}$$

$$y3x := \begin{bmatrix} \tanh(x3) \\ \tanh(x4) \end{bmatrix} \quad y3x = \begin{bmatrix} 0.457831 \\ 0.291313 \end{bmatrix}$$

$$I2 := -W2 \cdot y3x + G22 \cdot v3x \qquad I2 = \begin{bmatrix} 0.012446 \\ -0.010841 \end{bmatrix}$$

Now put all together as the "direct sum" of two 2x2 systems:

$$W := \begin{bmatrix} W1_{1,1} & W1_{1,2} & 0 & 0 \\ W1_{2,1} & W1_{2,2} & 0 & 0 \\ 0 & 0 & W2_{1,1} & W2_{1,2} \\ 0 & 0 & W2_{2,1} & W2_{2,2} \end{bmatrix} \qquad W = \begin{bmatrix} 2.15568 & -0.035206 & 0 & 0 \\ -0.035213 & 2.1522 & 0 & 0 \\ 0 & 0 & 2.15568 & -0.035206 \\ 0 & 0 & -0.035213 & 2.1522 \end{bmatrix}$$

$$I := \begin{bmatrix} I1_{1} \\ I1_{2} \\ I2_{1} \\ I2_{2} \end{bmatrix} \qquad I = \begin{bmatrix} 0.012446 \\ -0.010841 \\ 0.012446 \\ -0.010841 \end{bmatrix}$$

Finally check that Cdv/dt=0=g(v)=Wy(v)-Gv+I has the desired equlibrium points

First find the actual vectors from given ones plus variable ones

$$veq1 := vx1(x3, x4) \quad veq2 := vx2(x3, x4) \quad veq3 := vx3(x1, x2) \quad veq4 := vx4(x1, x2)$$

$$veq1 = \begin{bmatrix} 0.5 \\ 0.25 \\ 0.494563 \\ 0.3 \end{bmatrix} \quad veq2 = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.494563 \\ 0.3 \end{bmatrix} \quad veq3 = \begin{bmatrix} 0.494563 \\ 0.3 \\ 0.5 \\ 0.25 \end{bmatrix} \quad veq4 = \begin{bmatrix} 0.494563 \\ 0.3 \\ -0.5 \\ 0.5 \end{bmatrix}$$

$$vx := vx(x1, x2, x3, x4) \qquad vx = \begin{bmatrix} 0.494563 \\ 0.3 \\ 0.494563 \\ 0.3 \end{bmatrix}$$

Checking that g(veq_i)=0

$$g(v) := W \cdot y(v) - G \cdot v + I$$

$$g(veq1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad g(veq2) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad g(veq3) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad g(veq4) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$g(vx) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Note that vx has also become an equilibrium point

The neuron outputs for these actual inputs are

$$y(veq1) = \begin{bmatrix} 0.462117 \\ 0.244919 \\ 0.457831 \\ 0.291313 \end{bmatrix} \quad y(veq2) = \begin{bmatrix} -0.462117 \\ 0.462117 \\ 0.457831 \\ 0.291313 \end{bmatrix} \quad y(veq3) = \begin{bmatrix} 0.457831 \\ 0.291313 \\ 0.462117 \\ 0.244919 \end{bmatrix} \quad y(veq4) = \begin{bmatrix} 0.457831 \\ 0.291313 \\ -0.462117 \\ 0.462117 \end{bmatrix}$$

$$y(vx) = \begin{bmatrix} 0.457831 \\ 0.291313 \\ 0.457831 \\ 0.291313 \end{bmatrix}$$

This is z(x)=-0.5ln((1-z)/(1+z))

$$z(x) := -0.5 \cdot \ln\left[\frac{(1-x)}{(1+x)}\right] \quad n(x) := \begin{bmatrix} z(x_1) \\ z(x_2) \\ z(x_3) \\ z(x_4) \end{bmatrix}$$

To check n(y(v))=v

$$n(y(veq1)) = \begin{bmatrix} 0.5 \\ 0.25 \\ 0.494563 \\ 0.3 \end{bmatrix} \quad n(y(veq2)) = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.494563 \\ 0.3 \end{bmatrix}$$

$$n(y(veq3)) = \begin{bmatrix} 0.494563 \\ 0.3 \\ 0.5 \\ 0.25 \end{bmatrix} \quad n(y(veq4)) = \begin{bmatrix} 0.494563 \\ 0.3 \\ -0.5 \\ 0.5 \end{bmatrix} \quad n(y(vx)) = \begin{bmatrix} 0.494563 \\ 0.3 \\ 0.494563 \\ 0.3 \end{bmatrix}$$

These check our design. Note though that the original vectors v1, v2, v3, v4 are not equilibrium points, as seen by the following, but can be obtained by simple projections from the designed equilibria

$$g(v1) = \begin{bmatrix} 0 \\ 0 \\ 0.012446 \\ -0.010841 \end{bmatrix} \quad g(v2) = \begin{bmatrix} 0 \\ 0 \\ 0.012446 \\ -0.010841 \end{bmatrix} \quad g(v3) = \begin{bmatrix} 0.012446 \\ -0.010841 \\ 0 \\ 0 \end{bmatrix} \quad g(v4) = \begin{bmatrix} 0.012446 \\ -0.010841 \\ 0 \\ 0 \end{bmatrix}$$

Determination of the linear transformation to obtain the desired from the result:
Uaes [v1,v2,v3,v4][Veq1,veq2,veq3,veq4]^-1

$$Aeq := \begin{bmatrix} veq1_1 & veq2_1 & veq3_1 & veq4_1 \\ veq1_2 & veq2_2 & veq3_2 & veq4_2 \\ veq1_3 & veq2_3 & veq3_3 & veq4_3 \\ veq1_4 & veq2_4 & veq3_4 & veq4_4 \end{bmatrix}$$

$$Aeq^{-1} = \begin{bmatrix} -1.555038 & -10.220152 & 2.55674 & 10.226959 \\ -0.858307 & 0.566771 & 0.169637 & 0.678549 \\ 2.55674 & 10.226959 & -1.555038 & -10.220152 \\ 0.169637 & 0.678549 & -0.858307 & 0.566771 \end{bmatrix}$$

$$A := \begin{bmatrix} v1_1 & v2_1 & v3_1 & v4_1 \\ v1_2 & v2_2 & v3_2 & v4_2 \\ v1_3 & v2_3 & v3_3 & v4_3 \\ v1_4 & v2_4 & v3_4 & v4_4 \end{bmatrix} \qquad A = \begin{bmatrix} 0.5 & -0.5 & 0 & 0 \\ 0.25 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix}$$

$$Xconvert := A \cdot Aeq^{-1} \qquad Xconvert = \begin{bmatrix} -0.348365 & -5.393461 & 1.193551 & 4.774205 \\ -0.817913 & -2.271652 & 0.724004 & 2.896014 \\ 1.193551 & 4.774205 & -0.348365 & -5.393461 \\ 0.724004 & 2.896014 & -0.817913 & -2.271652 \end{bmatrix}$$

As a check we should get v1,v2,v3,v4 back

$$Xconvert \cdot veq1 = \begin{bmatrix} 0.5 \\ 0.25 \\ 0 \\ 0 \end{bmatrix} \quad Xconvert \cdot veq2 = \begin{bmatrix} -0.5 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} \quad Xconvert \cdot veq3 = \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0.25 \end{bmatrix}$$

$$Xconvert \cdot vx = \begin{bmatrix} 0.232221 \\ 0.140864 \\ 0.232221 \\ 0.140864 \end{bmatrix} \quad Xconvert \cdot veq4 = \begin{bmatrix} 0 \\ 0 \\ -0.5 \\ 0.5 \end{bmatrix}$$