EE434
02/22/06

$$C\frac{dn}{dt} = +Wa - Gn + I_b \quad ; \quad n(0) \text{ given}$$

$$a = f(n)$$

n = n-vector of capacitor voltages
a = n-vector of amplifier current outputs of voltage controlled devices

store data in equilibria points:
need this to be stable for useful equilibria
Set up a Lyapunov function $\equiv$ energy

Equilibria: $\frac{dn}{dt} = 0 \Rightarrow 0 = Wa - Gn + I_B$

$$= Wf(n) - Gn + I_B$$

desire the n's at equilibria; normally give these and design with I_B & W (assuming G known)

Let S = # of neurons (p. 18-8); 0 = zero S-vector

If subtract at 2 different n's eliminates I_B
n_1 & n_2

$$0 = Wf(n_1) - Gn_1 + I_B \quad \leftarrow \text{gives } I_B \text{ if know } W$$

$$- \quad 0 = Wf(n_2) - Gn_2 + B_B$$

$$0 = W(f(n_1) - f(n_2)) - G(n_1 - n_2)$$

use differences of $S+1$ equilibria

get eqs: $G(n_1 - n_2) = W(f(n_1) - f(n_2))$

$G(n_1 - n_3) = W(f(n_1) - f(n_3))$

$\vdots \qquad \vdots$

$G(n_1 - n_{S+1}) = W(f(n_1) - f(n_{S+1}))$

rewrite as columns

$$\left[G(n_1 - n_2) \quad G(n_1 - n_3) \; \cdots \; G(n_1 - n_{S+1}) \right] = \text{an } S \times S \text{ matrix}$$

$\underset{\shortparallel}{}$ s-vector = Equilibria points

$S \times S$ matrix of input conductances

$$= \underbrace{W}_{\substack{S\times S \\ \text{matrix} \\ \text{of weight}}} \underbrace{\left[(f(n_1) - f(n_2)) \; (f(n_1) - f(n_3)) \; \cdots \; (f(n_1) - f(n_{S+1})) \right]}_{\substack{S \times S \text{ matrix of activation} \\ \text{s- vector function differences}}}$$

$\underset{\shortparallel}{X}$

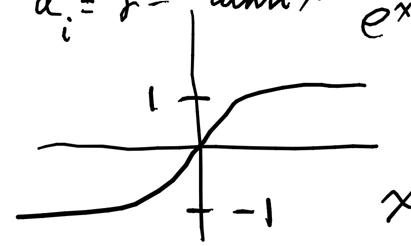
W is the unknown, everything else is known

$$Y = WX \Longrightarrow W = YX^{-1}$$

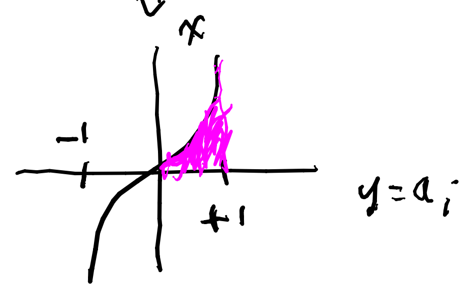
Energy function: $V(\alpha)$; eq. (18.8)

$$V(a) = -\frac{1}{2} a^T W a + \sum_{i=1}^{S} G_i \int_0^{a_i} f_i^{-1}(u_i)\, du_i - a^T I_b \qquad (18.8)$$

$$a_i = y = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



$\Downarrow$



V(a) is bounded below

look at $\frac{dV(a)}{dt}$; this will be ≤ 0

$$\frac{dV(a)}{dt} = -\frac{1}{2} \frac{da^T}{dt}(Wa) - \frac{1}{2} a^T W \frac{da}{dt}$$

$$+ \sum_{i=1}^{S} G_i\, f_i^{-1}(a_i) \frac{da_i}{dt} - \frac{da^T}{dt} \cdot I_B$$

$$= -\frac{1}{2} a^T W^T \frac{da}{dt} + n^T G^T \frac{da}{dt} - I_B^T \frac{da}{dt} - \frac{1}{2} a^T W \frac{da}{dt}$$

$$= \left\{ -\frac{1}{2} a^T [W^T + W] + n^T G^T - I_B^T \right\} \frac{da}{dt}$$

note $n = f(a)$, $a = f^{-1}(n)$; $\frac{da}{dt} = \underbrace{\frac{df^{-1}}{dn}}_{\text{square}} \cdot \frac{dn}{dt}$

$$C\dot{n} = Wa - Gn + I_B$$

square will have entries >0 on diagonal here

$$\frac{dV(\alpha)}{dt} = \left\{ -\alpha^T [W^T + W^T] - \frac{1}{2}\alpha^T (W - W^T) + n^T C^T - I_B^T \right\} \left(\frac{df^{-1}}{dn}\right) \frac{dn}{dt}$$

$$= -\frac{dn}{dt}^T \cdot C^{-1} \left\{ 1 - C \frac{1}{2} \alpha^T (W - W^T) \right\} \left(\frac{df^{-1}}{dn}\right) \cdot \frac{dn}{dt}$$

$$= \underbrace{-\frac{dn}{dt}^T \underbrace{C^{-1} \cdot \left(\frac{df^{-1}}{dn}\right)}_{\text{diagonal \& positive definite}} \cdot \frac{dn}{dt}}_{\leq 0} + \frac{dn}{dt}^T C \underbrace{\alpha^T (W - W^T)}_{\text{if } W = W^T \Rightarrow 0} \frac{df^{-1}}{dn} \cdot \frac{dn}{dt}$$

$\Rightarrow$ stable equilibria if $W = W^T$ & $\dfrac{df(n)}{dn} > 0$

& $C > 0$