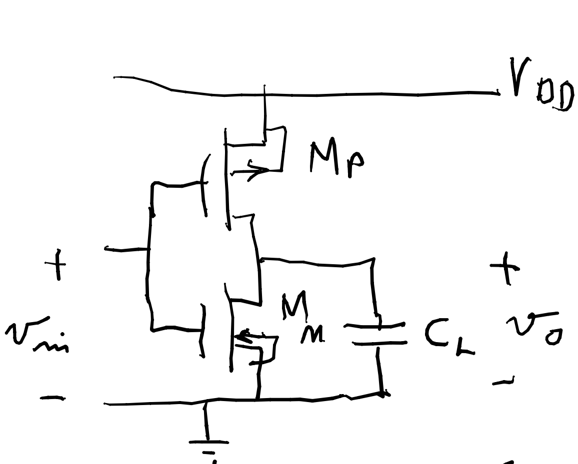
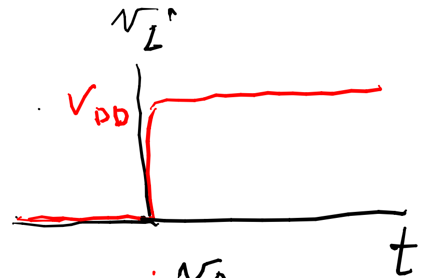
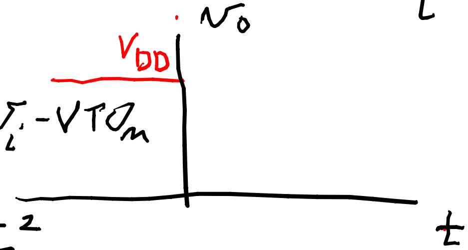
EE 303
03/10/06

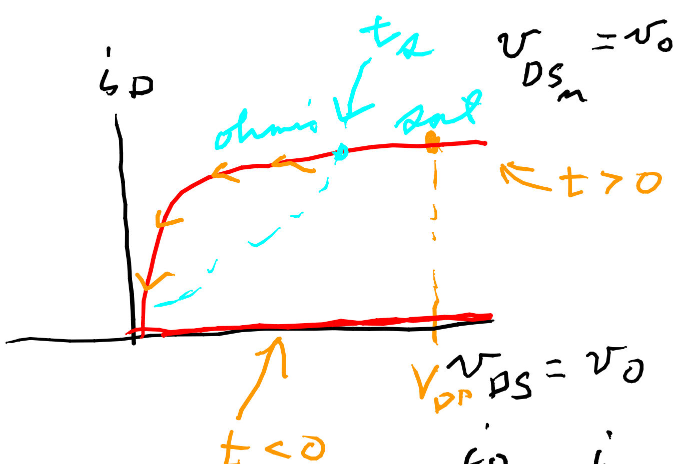






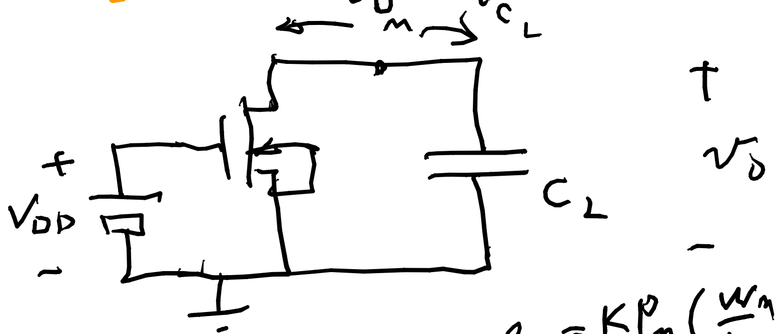
$$i_{D_n} = \frac{KP_n}{2}\left(\frac{W_n}{L_n}\right)\begin{cases}(V_{GS_n}-VTO_n)^2 & v_O \geq v_I - VTO_n \\ 2(V_{GS_n}-VTO_n)v_O - v_O^2 & \end{cases}$$

$$v_{DS_n} = v_O \lesssim v_{GS_n} - VTO_n = V_{DD} - VTO_n$$



let t_a = time when go from sat. to ohmic

Ep.



$v_O(0_+) = V_{DD}$

$$\beta_n = \frac{KP_n}{2}\left(\frac{W_n}{L_n}\right)$$

KCL: $-i_D - i_{C_L} = 0$

$t < t_a$ $\quad i_{D_n} = \beta_n \cdot (V_{DD} - VTO_n)^2 = -i_{C_L} = -C_L\frac{dv_O}{dt}$

$$\dot{v}_o = -\frac{\beta_n}{C_L}(V_{DD}-VTO_n)^2 = \text{constant} \Rightarrow \int_0^t \dot{v}_o\,dt = \int_0^t \text{constant}$$

$$v_o(t) = -\frac{\beta_n}{C_L}(V_{DD}-VTO_n)^2 \cdot \tau \Big|_0^t \, , \qquad 0 \le t \le t_s$$

t_s is when $v_o = v_i - VTO_n = V_{DD} - VTO_n$

$$v_o(t) = -\left(\frac{\beta_n}{C_L}\right)(V_{DD}-VTO_n)^2 t + V_{DD} \quad = v_o(0_+) \qquad 0<t<t_s$$

$$t_s: \quad V_{DD}-VTO_n = -\left(\frac{\beta_n}{C_L}\right)(V_{DD}-VTO_n)^2 t_s + V_{DD}$$

$$t_s = \frac{VTO_n}{\left(\frac{\beta_n}{C_L}\right)(V_{DD}-VTO_n)^2}$$

for $t > t_s$

$$-C_L \frac{dv_o}{dt} = \beta_n\left(2(V_{DD}-VTO_n)v_o - v_o^2\right)$$

$$\dot{v}_o = -\frac{\beta_n}{C_L}\left[2(V_{DD}-VTO_n)v_o - v_o^2\right] \quad \text{a Riccati}$$

$$\frac{dv_o}{-\left(\frac{\beta_n}{C_L}\right)\left[2(V_{DD}-VTO_n)v_o - v_o^2\right]} = dt \qquad \text{let } \alpha = \beta_n/C_L, \quad \gamma = V_{DD}-VTO_n$$

$$\int_{t_s}^{t} d\tau = \int_{v_o(t_s)}^{v(t)} \frac{dv_o}{\alpha v_o(v_o-2\gamma)} = \frac{1}{\alpha}\int_{v_{os}}^{v_o(t)}\left[\frac{k_1}{v_o} + \frac{k_2}{v_o-2\gamma}\right]dv_o$$

$$k_1(v_0 - 2\gamma) + k_2 v_0 = 1 \quad \Rightarrow \quad k_1 + k_2 = 0 \Rightarrow k_2 = -k_1$$

$$-2\gamma k_1 = 1 \Rightarrow k_1 = -1/2\gamma, \quad k_2 = 1/2\gamma$$

$$t - t_s = \frac{1}{\alpha}\left[k_1 \ln v_0 - k_1 \ln(v_0 - 2\gamma)\right]\Bigg|_{v_0(t_s)}^{v(t)}$$

$$= \frac{-1}{2\gamma\alpha}\left[\ln\left(\frac{v_0}{v_0 - 2\gamma}\right)\right]\Bigg|_{v_0(t_s)}^{v_0(t)} \quad ; \quad v_0(t_s) = V_{DD} - VTO_n = \gamma$$

$$= \frac{-1}{2\alpha\gamma}\left[\ln\left(\frac{\frac{v_0(t)}{v_0(t) - 2\gamma}}{\frac{\gamma}{\gamma - 2\gamma}}\right)\right]$$

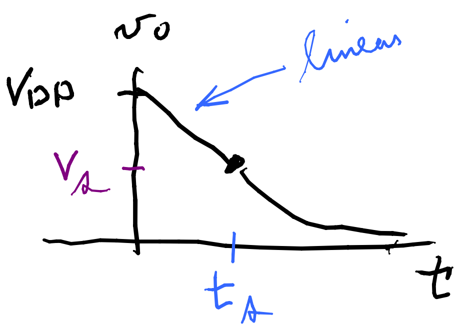
$$= -\frac{1}{2\alpha\gamma}\ln\left(\frac{v_0/\gamma}{\frac{2\gamma - v_0}{\gamma}}\right) = \frac{-1}{2\gamma\alpha}\ln\left(\frac{v_0/\gamma}{2 - \frac{v_0}{\gamma}}\right)$$

$$\therefore \; e^{-2\gamma\alpha(t - t_s)} = \left(\frac{v_0/\gamma}{2 - v_0/\gamma}\right) \Rightarrow e^p = \frac{x}{2 - x} \Rightarrow 2e^p - e^p x = x$$

$$x(1 + e^p) = 2e^p$$

$$\frac{v_0(t)}{\gamma} = \frac{2e^{-2\gamma\alpha(t - t_s)}}{1 + e^{-2\gamma\alpha(t - t_s)}} \qquad t \geq t_s$$

@ $t = t_s$, $e^0 = 1$



gives $v_0(t) = \frac{2}{2}\gamma = \gamma = V_{DD} - VTO_n$

call $\gamma = V_s$

$$v_0(t) = \frac{2V_s}{1 + e^{+2V_s \cdot \frac{\beta}{C_L}(t - t_s)}}, \quad t > t_s$$

More added: delay time: $\Rightarrow t$ for $v_d = V_{DD}/2$

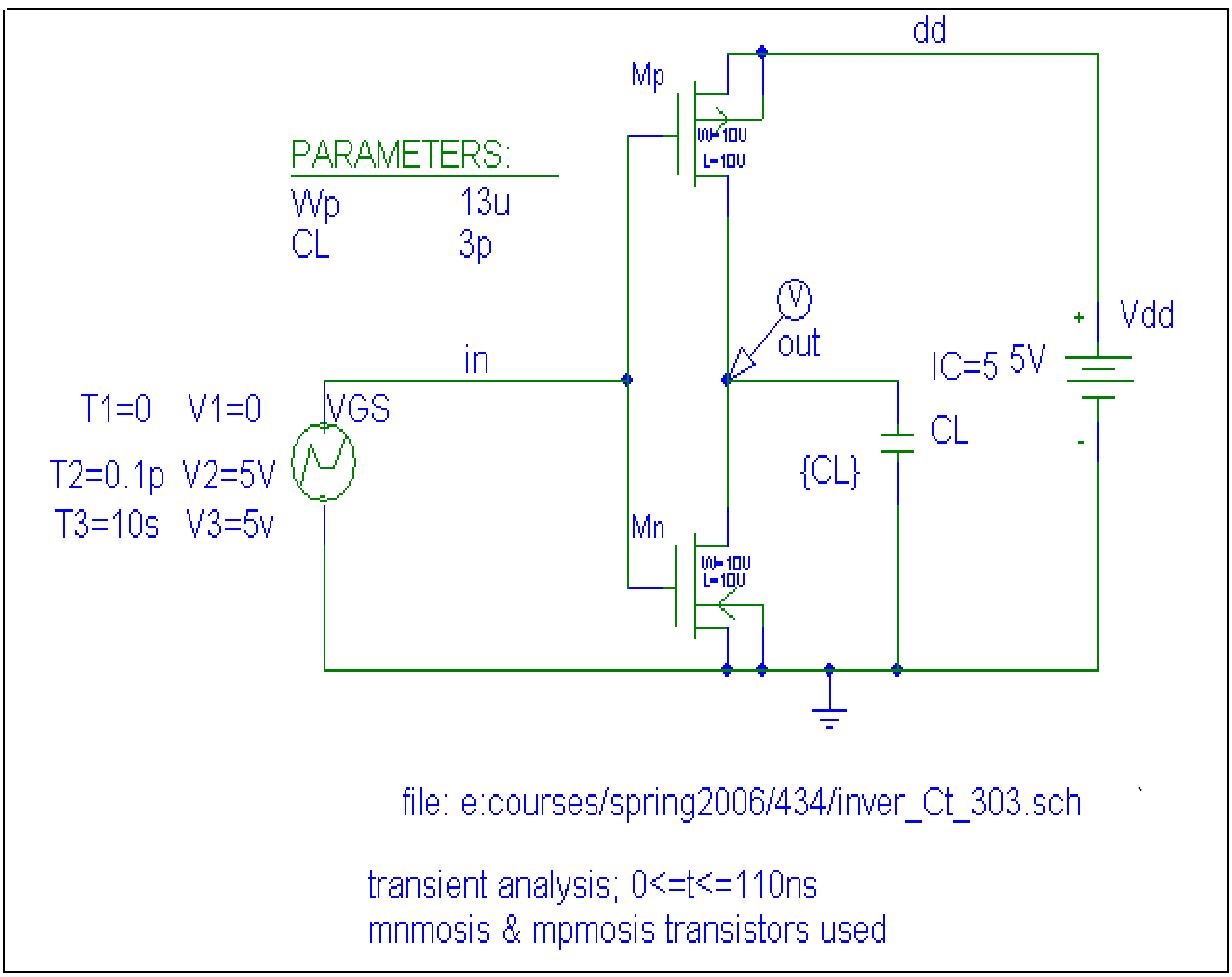
from above

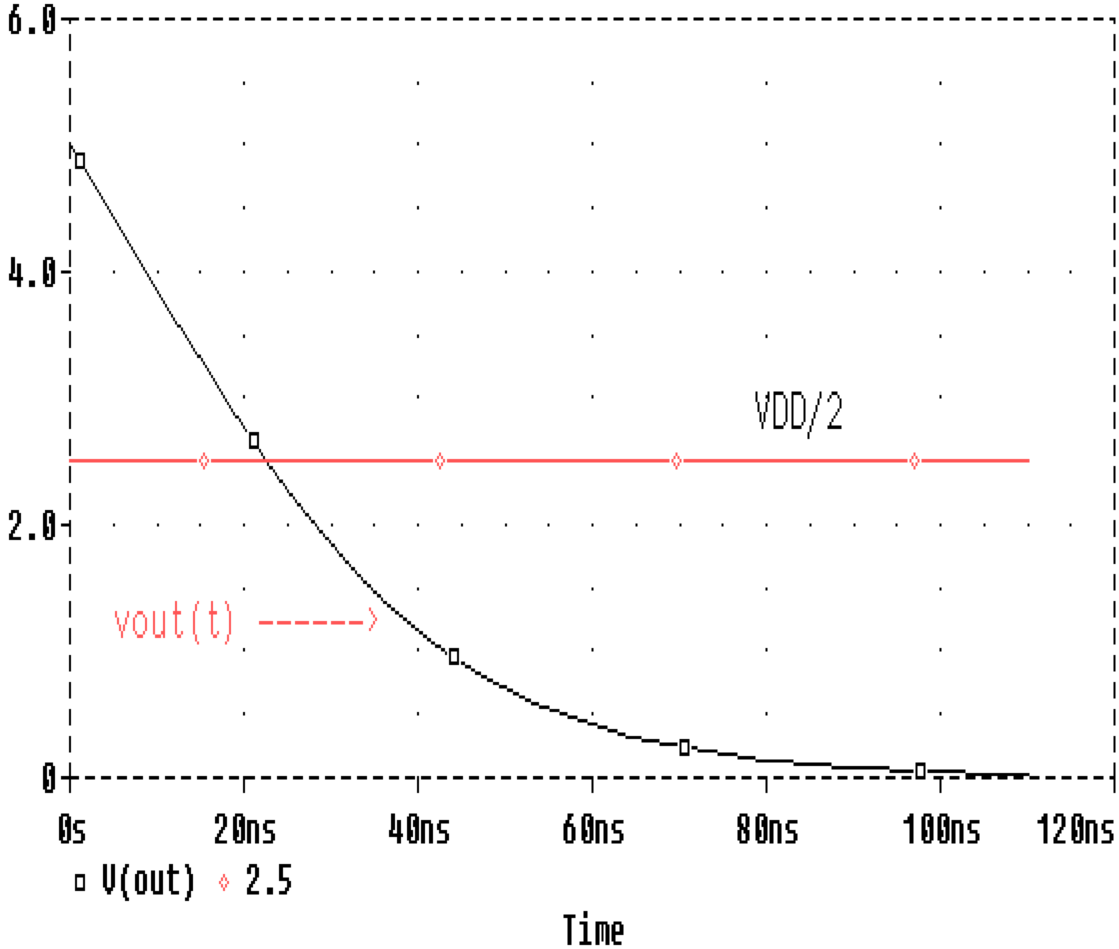
$$t = t_s - \left(\frac{C_L}{\beta}\right)\cdot\frac{1}{2V_s}\ln\left(\frac{v_0/V_s}{2 - v_0/V_s}\right) \qquad \text{if } t_d > t_s$$

or

$$t_d = t_s - \left(\frac{C_L}{\beta}\right)\frac{1}{2V_s}\cdot\ln\left(\frac{V_{DD}/V_s}{4 - V_{DD}/V_s}\right) \qquad \text{if } t_d > t_s$$

[if $t_d < t_s$ use $t = \dfrac{V_{DD} - v_0(t)}{\left(\beta/C_L\right) V_s^2}$ or $t_d = \left(\dfrac{C_L}{\beta}\right)\dfrac{V_{DD}}{2V_s^2}$




6.0
4.0
2.0
0
VDD/2
vout(t) ------->
0s
20ns
40ns
60ns
80ns
100ns
120ns
V(out)
2.5
Time