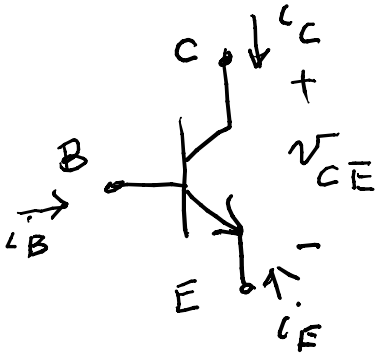
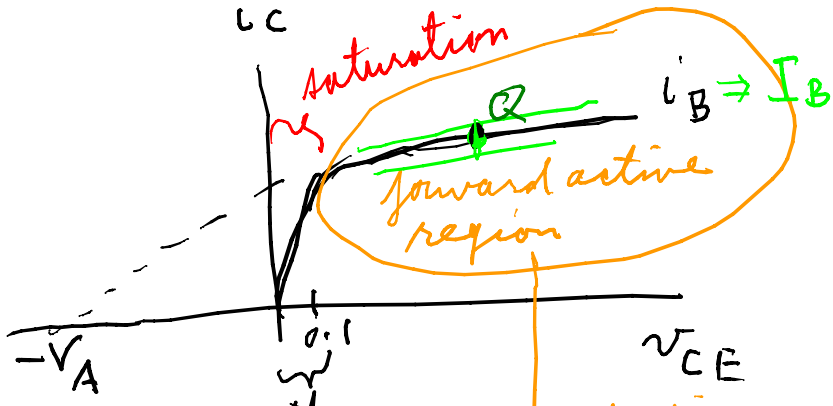
EE303
02/22/06

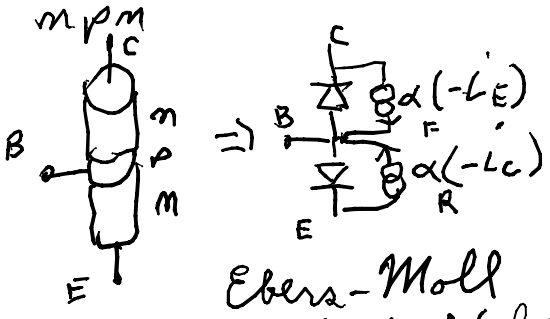
Hybrid-Π model

BJT

p. 448







Ebers-Moll equivalent (large signal)

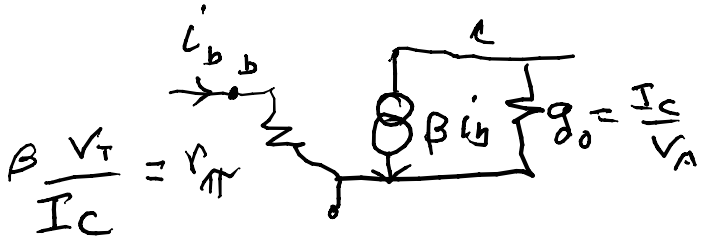
Small signal

$$i_c = f(i_B, v_{CE}) = I_C + \frac{\partial i_C}{\partial i_B}(i_B - I_B) + \frac{\partial i_C}{\partial v_{CE}}(v_{CE} - V_{CE})$$

$$i_c = i_C - I_C = \left.\frac{\partial i_C}{\partial i_B}\right|_Q i_b + \left.\frac{\partial i_C}{\partial v_{CE}}\right|_Q v_{ce}$$

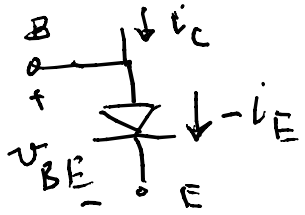
$+ \cdots$ ignore for small signal

$$= \beta i_b + g_o v_{ce} \qquad \text{here } g_o \approx \frac{I_C}{V_A}$$



$i = yv$

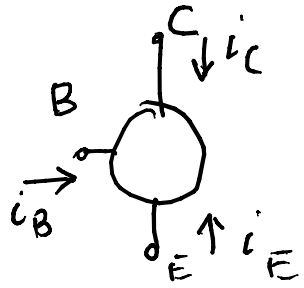
look



$$-i_E = I_{SE} e^{v_{BE}/V_T}, \quad V_T = \frac{kT}{|q|}$$

$$\left.\frac{d(-i_E)}{dv_{BE}}\right|_Q = \frac{I_{SE} e^{v_{BE}/V_T}}{V_T}\Bigg|_Q$$

$$= \frac{-I_E}{V_T} \Rightarrow i_e = \frac{-I_E}{V_T} \cdot v_{be}$$



KCL: $i_B + i_C + i_E = 0 \equiv i_b + i_c + i_e = 0$

if $i_c = \beta i_b \Rightarrow i_b + \beta i_b = -i_e \Rightarrow i_e = -(1+\beta) i_b$

$$i_c = \beta i_b = -\frac{\beta}{1+\beta} i_e = -\alpha i_e$$

$$\alpha = \frac{\beta}{1+\beta}; \Rightarrow (1+\beta)\alpha = \beta$$

$$\beta - \beta\alpha = \alpha$$

$$\beta = \frac{\alpha}{1-\alpha}$$

numerical:

$\beta = 50 \Rightarrow \alpha = \frac{50}{51} \approx 0.98$

$$-i_e = \frac{|I_E|}{V_T} \cdot v_{be}$$

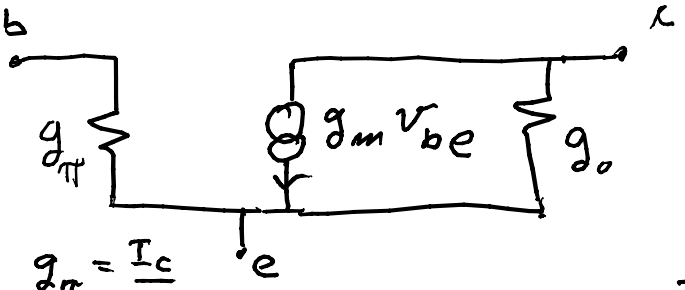
$$(1+\beta) i_b = \frac{|I_E|}{V_T} \cdot v_{be}$$

$$i_b = \frac{|I_E|}{(1+\beta)V_T} \cdot v_{be}$$

$$g_\pi = \frac{|I_E|}{(1+\beta)V_T} = \frac{1}{\alpha} \cdot \frac{I_C}{(1+\beta)V_T}$$

$$= \frac{I_C}{\beta V_T} = g_\pi$$

$$\beta \cdot i_b = \beta\left(g_\pi v_{be}\right) = \frac{\beta I_C}{\beta V_T} \cdot v_{be} = \frac{I_C}{V_T} \cdot v_{be} = g_m v_{be}$$



$g_\pi = \frac{I_C}{\beta V_T} = \frac{1}{\beta} g_m$

$g_m = \frac{I_C}{V_T}$

$g_o = \frac{I_C}{V_A} = \frac{V_T}{V_A} \cdot \frac{I_C}{V_T} = \eta_A \cdot g_m$