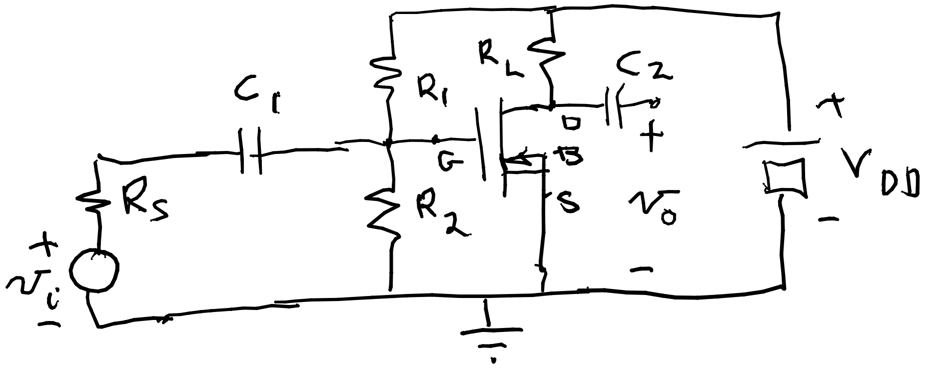
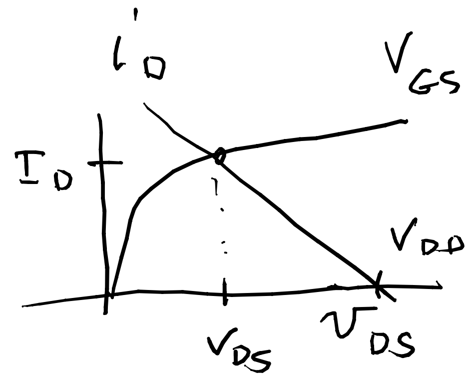
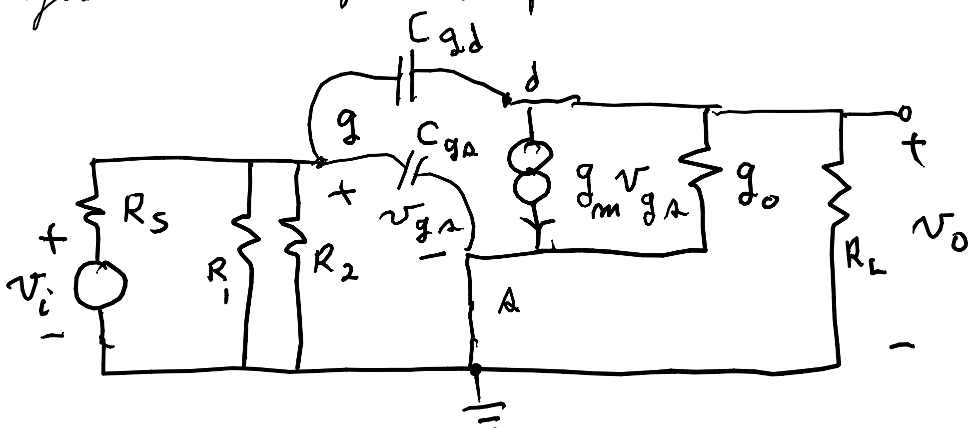
EE303
02/15/06





for small signals; equivalent circuit



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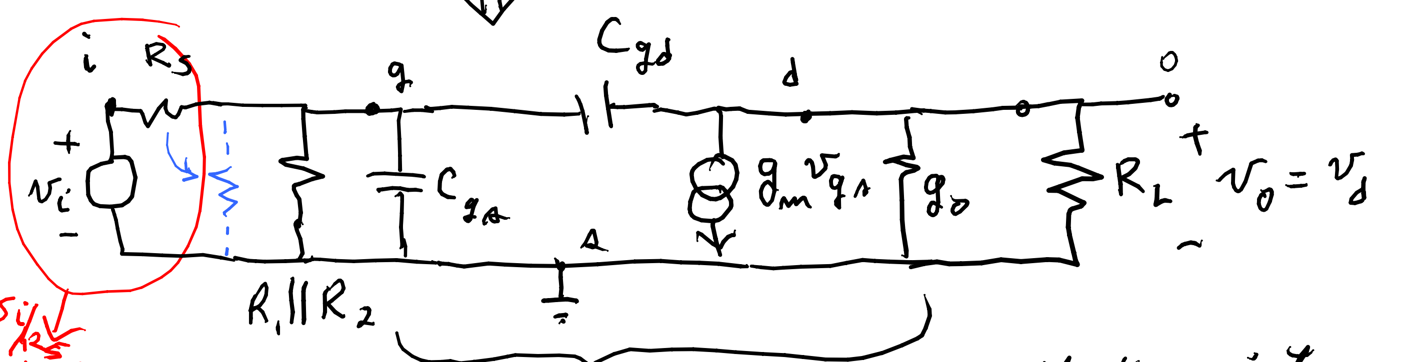
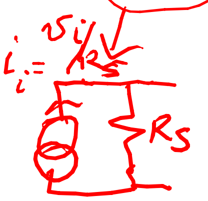




π equivalent circuit for the transistor

as freq → ∞, the capacitors become shorts

$v_o \to 0$

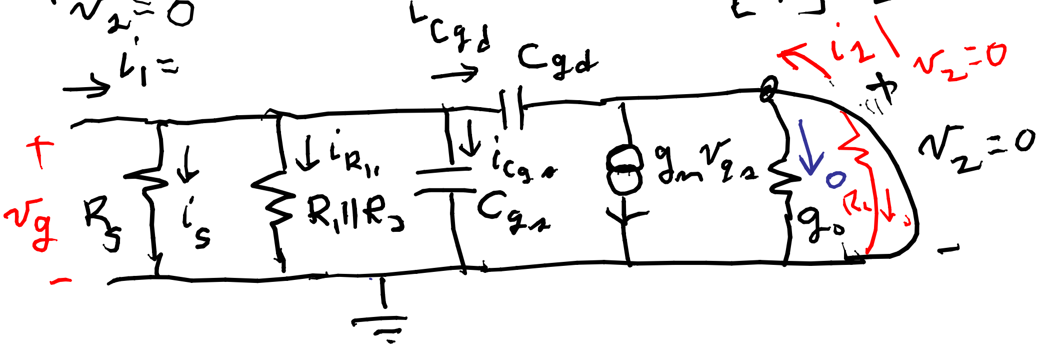
$$\frac{v_o}{v_i}(s) = T(s) = \frac{m_1 s + m_0}{s^2 + d_1 s + d_0} \; ; \quad T(0) = \text{low frequency gain} = \frac{m_0}{d_0}$$

nodal admittance $i_{node} = Y_{node} \cdot v_{node}$

$$i_{node} = \begin{bmatrix} i_i \\ 0 \end{bmatrix}, \quad v_{node} = \begin{bmatrix} v_g \\ v_d \end{bmatrix} \quad Y_{node} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2 = 0}$$



$$i_1 = (G_s + G_{/\!/} + sC_{gs} + sC_{gd})\, v_g \Rightarrow y_{11} = G_s + G_{/\!/} + s(C_{gs} + C_{gd})$$

$$= G_{in} + s(C_{gs} + C_{gd})$$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \; ; \quad i_2\Big|_{v_2=0} = g_m v_{gs} + (-i_{C_{gd}}) = (g_m - sC_{gd})\, v_g \; ; \; y_{21} = g_m - sC_{gd}$$

$$\frac{v_o}{i_i} = \frac{v_o}{v_i/R_s} = R_s \frac{v_o}{v_i} \qquad \begin{bmatrix} i_i \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_g \\ v_d \end{bmatrix} \quad v_d = v_o$$

here (2nd eq) $0 = y_{21} v_g + y_{22} \cdot v_o \Rightarrow v_g = -\dfrac{y_{22}}{y_{21}} \cdot v_o$

also from 1st eq.

$$i_i' = y_{11} v_g + y_{12} v_o = \left[y_{11}\left(-\frac{y_{22}}{y_{21}}\right) + y_{12} \right] v_o = -\frac{\det Y}{y_{21}} \cdot v_o$$

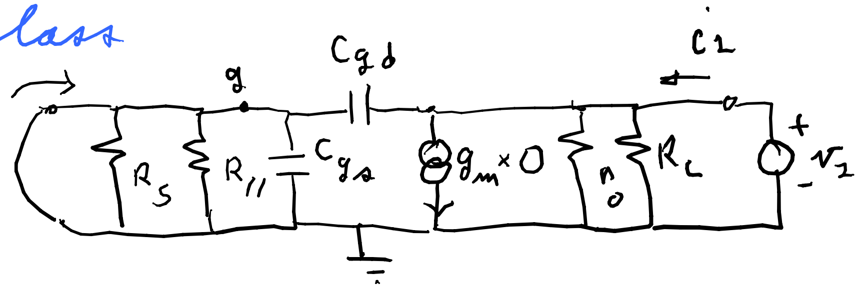
$$\downarrow$$

$$\frac{v_o}{i_i'} = \frac{y_{21}}{-\det Y_{nodal}} = -\left(\frac{g_m - sC_{gd}}{\det Y_{nodal}} \right) \qquad \det Y = y_{11}y_{22} - y_{12}y_{21}$$

Notes added after class

$$y_{22} = \left. \frac{i_2}{v_2} \right|_{v_1 = 0}$$

$\Rightarrow$ short at port 1 = g



$$= g_o + G_L + sC_{gd} = G_o + sC_{gd}$$

$$y_{12} = \left. \frac{i_1}{v_2} \right|_{v_1 = 0} = \frac{\text{current in } C_{gd} \text{ (left to right)}}{\text{voltage on } C_{gd} \text{ (+ on right, − on left)}} = -sC_{gd}$$

[as no current in R_S, R_{11} or C_{gs} with 0 volts on them]

$\therefore$

$$Y_{nodal}(s) = \begin{bmatrix} G_{in} + s(C_{gs} + C_{gd}) & -sC_{gd} \\ g_m - sC_{gd} & G_o + sC_{gd} \end{bmatrix}$$

where $G_{in} = G_S + G_{11}$, $G_o = g_o + G_L$

and

$$\det Y = y_{11}y_{22} - y_{12}y_{21} = [G_{in} + s(C_{gs} + C_{gd})][G_o + sC_{gd}] - (-sC_{gd})(g_m - sC_{gd})$$

$$= G_{in}G_o + s[G_{in}C_{gs} + G_o(C_{gs} + C_{gd}) + g_m C_{gd}] + s^2 C_{gs}C_{gd}$$

$\therefore$

$$\frac{v_o}{v_i} = \frac{1}{R_S}\frac{v_o}{i_i'} = \frac{1}{R_S} \cdot \frac{-y_{21}}{\det Y} = T(s)$$

$$= \frac{-g_m + sC_{gd}}{R_S\left(C_{gs}C_{gd}s^2 + \left[G_{in}C_{gs} + G_o(C_{gs}+C_{gd}) + g_mC_{gd}\right]s + G_{in}G_o\right)}$$

as a check, at DC where $s=0$

$$T(0) = \frac{-g_m}{R_S \cdot G_{in}G_o} \quad \text{and if } g_o << G_L,\ G_{//} << G_S$$

$$= \frac{-g_m}{R_S \cdot \frac{1}{R_S} \cdot \frac{1}{R_L}} = -g_m R_L$$