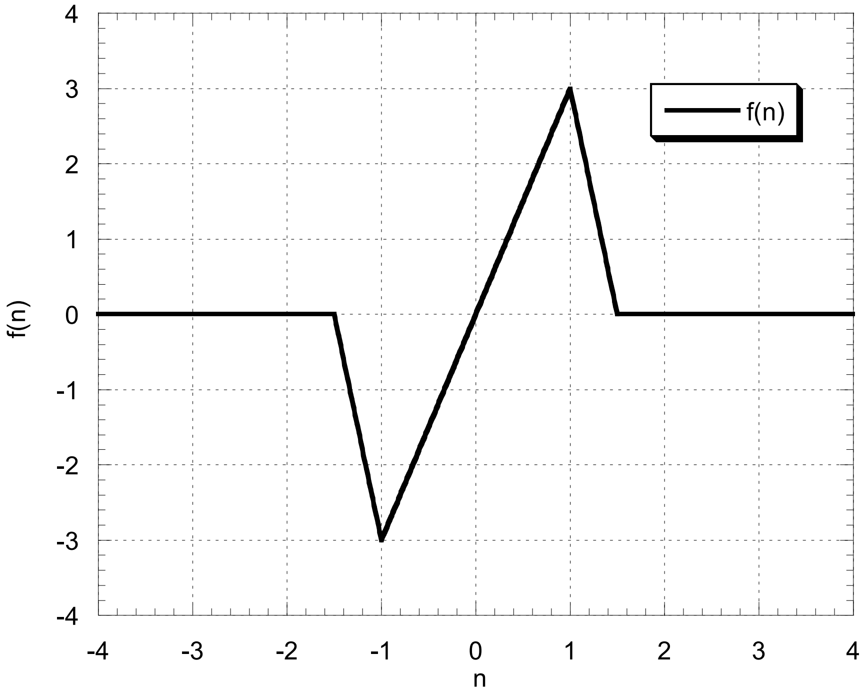
ENEE434 Spring 2005
Homework 6 Solution: Y.J. 04/14/05

Problem 1: (30)

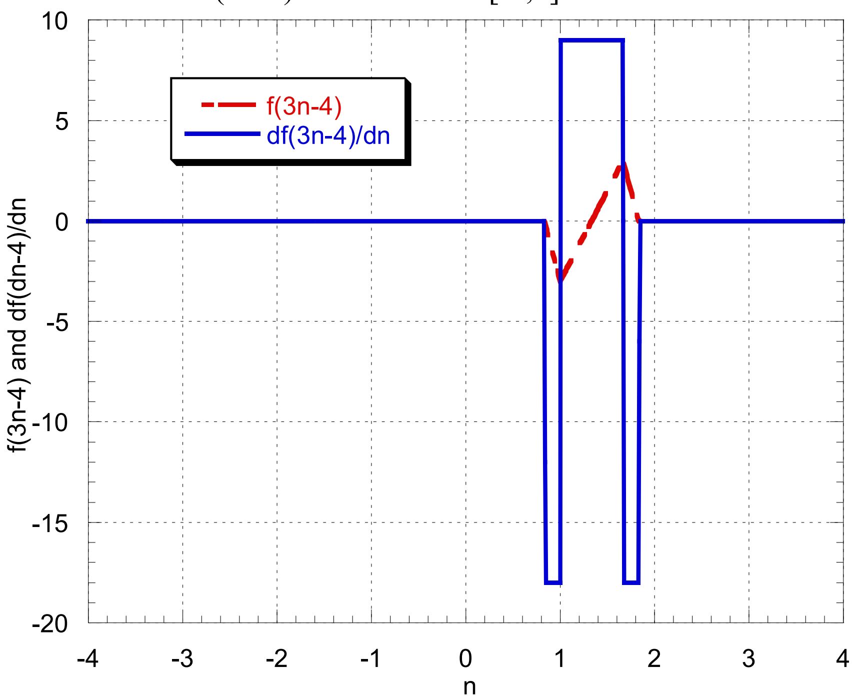
$f(n) = 3[-|n+1.5| + 1.5|n+1| - 1.5|n-1| + |n-1.5|]$

a) sketch f(n) in the interval [-4, 4]:



Important transition points: (-1.5, 0), (-1, -3), (1, 3), and (1.5, 0).

b) sketch the derivative of f(3n-4) in the interval [-4,4]:



Important transition points:
(5/6, 0 or -18), (1, -18 or 9), (5/3, 9 or -18) and (11/6, -18 or 0).

c) Write f(3n-4) as a weighted sum of shifted satlins functions:

Since $satlins = \frac{1}{2} \cdot \left[|n+1| - |n-1| \right]$,

$$f(n) = 3\left[-|n+1.5| + 1.5|n+1| - 1.5|n-1| + |n-1.5| \right]$$

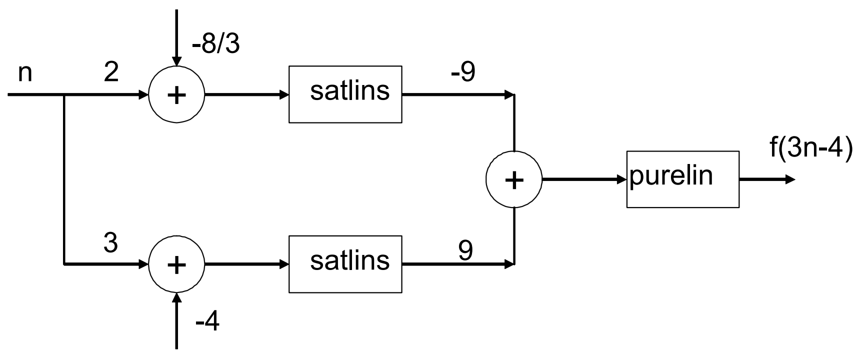
$$= -9 \cdot \left[\frac{1}{2} \cdot \left(\left| \frac{2}{3}n + 1 \right| - \left| \frac{2}{3}n - 1 \right| \right) \right] + 9 \cdot \left[\frac{1}{2} \cdot \left(|n+1| - |n-1| \right) \right]$$

Therefore:

$$f(n) = -9 \cdot \left[satlins\left(\frac{2}{3}n \right) - satlins(n) \right] \text{, and}$$

$$f(3n-4) = -9 \cdot \left[satlins\left(2n - \frac{8}{3} \right) - satlins(3n-4) \right]$$

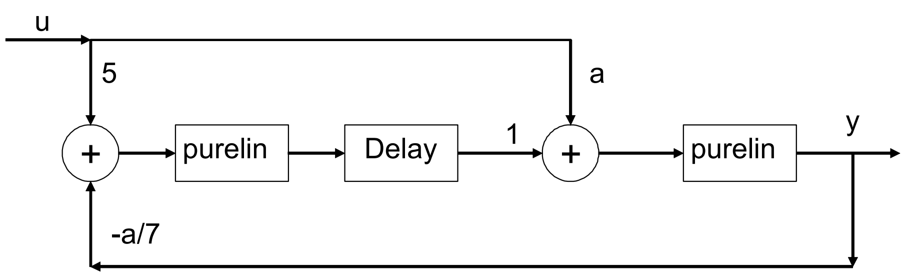
d) Draw a neural network which realizes f(3n-4):



Problem 2: (35)

A digital filter: $\dfrac{y(z)}{u(z)} = \dfrac{a + 5Z^{-1}}{1 + (1/7)az^{-1}}$

a) Neural network realization:



b) Unit step function response:

$$y(t) = au(t) + 5u(t-1) - (a/7)y(t-1)$$

$$y(0) = 0$$

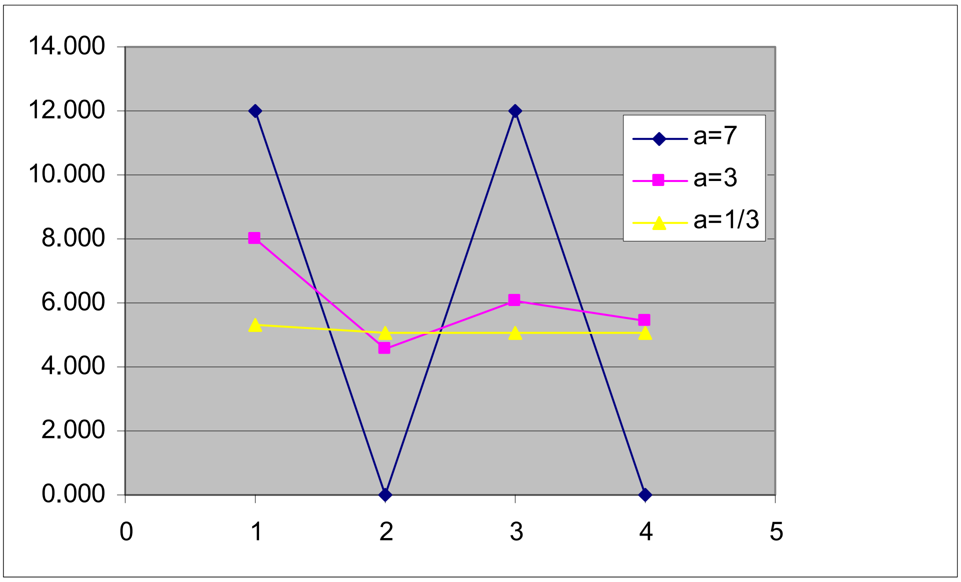
$$y(1) = a \cdot 1 + 5 \cdot 1 - (a/7) \cdot 0 = a + 5$$

$$y(2) = a \cdot 1 + 5 \cdot 1 - (a/7) \cdot (a+5) = (1 - a/7) \cdot (a+5)$$

$$y(3) = a \cdot 1 + 5 \cdot 1 - (a/7) \cdot \left[(1 - a/7) \cdot (a+5) \right] = \left[1 - (a/7) \cdot (1 - a/7) \right] \cdot (a+5)$$

$$y(4) = a \cdot 1 + 5 \cdot 1 - (a/7) \cdot \left\{ \left[1 - (a/7) \cdot (1 - a/7) \right] \cdot (a+5) \right\} = \left\{ 1 - (a/7) \cdot \left[1 - (a/7) \cdot (1 - a/7) \right] \right\} \cdot (a+5)$$

t	u(t)	y(t, a)	y(t), a=7	y(t), a=3	y(t), a=1/3
0	1	0	0	0	0
1	1	a+5	12	8	5.333
2	1	(a+5)*(1-a/7)	0	4.571	5.079
3	1	(a+5)*[1-(a/7)*(1-a/7)]	12	6.041	5.091
4	1	(a+5)*{1-(a/7)*[1-(a/7)*(1-a/7)]}	0	5.411	5.091



c) (RWN) By using the time input to be n, kept in increments under n=0.1, and the training output to be f(3n-4), the network of a can be trained by adjusting a. Since there is only one parameter, a, to adjust, the error can only be set to 0 at one value of n.

Problem 3: (30)

a) Calculate equilibrium points in terms of generic, W, b, ε.

$$\varepsilon \frac{dn(t)}{dt} = -n(t) + W \cdot a(t) + b \qquad \text{(Equation 18.6 from text book, pp. 18-4)}$$

$$a(t) = f(n(t)) \qquad \text{(Equation 18.7 from text book, pp. 18-4)}$$

At equilibrium point, $\frac{dn(t)}{dt} = 0$. Therefore $-n(t) + W \cdot a(t) + b = 0$.

with $a(t) = f(n(t)) = purelin(5n(t)) = 5n(t)$,

$-n(t) + W \cdot a(t) + b = -n(t) - W \cdot 5 \cdot n(t) + b = 0$,

at equilibrium, we have:

$$\begin{cases} n(t) = (I - 5W)^{-1} \cdot b \\ a(t) = 5 \cdot (I - 5W)^{-1} \cdot b \end{cases}$$

b) (RWN) This no longer needs to be a valid Lyapunov function since it needs not be bounded below since f(.) does not saturate, even though dV(a)/dt<0 will hold when $W=W^T$.

c) $\frac{1}{3} \cdot \frac{dn}{dt} = -5n + \begin{bmatrix} 2 & -3p \\ p & -1 \end{bmatrix} a + \begin{bmatrix} 5 \\ -9 \end{bmatrix}, \qquad n = \begin{bmatrix} n1 \\ n2, \end{bmatrix}, \qquad a = \begin{bmatrix} 5n1 \\ 5n2 \end{bmatrix}$

At equilibrium, dn/dt=0,

$$-5\cdot\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}n1\\n2\end{bmatrix}+\begin{bmatrix}2 & -3p\\p & -1\end{bmatrix}\cdot\begin{bmatrix}5n1\\5n2\end{bmatrix}+\begin{bmatrix}5\\-9\end{bmatrix}=0,$$

$$-\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}n1\\n2\end{bmatrix}+\begin{bmatrix}2 & -3p\\p & -1\end{bmatrix}\cdot\begin{bmatrix}n1\\n2\end{bmatrix}+\begin{bmatrix}1\\-9/5\end{bmatrix}=0,$$

$$\begin{bmatrix}n1\\n2\end{bmatrix}=\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}2 & -3p\\p & -1\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}1\\-9/5\end{bmatrix}$$

$$=\left(\begin{bmatrix}-1 & 3p\\-p & 2\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}1\\-9/5\end{bmatrix}=\frac{1}{3p^2-2}\cdot\begin{bmatrix}2 & -3p\\p & -1\end{bmatrix}\cdot\begin{bmatrix}1\\-9/5\end{bmatrix}=\begin{bmatrix}\frac{5.4p+2}{3p^2-2}\\\frac{p+1.8}{3p^2-2}\end{bmatrix}$$

Therefore, equilibrium point: $n=\begin{bmatrix}\frac{5.4p+2}{3p^2-2}\\\frac{p+1.8}{3p^2-2}\end{bmatrix}, \quad a=5n.$

c) To look at the stability we look at zeros of the characteristic polynomial formed from
$s\cdot\varepsilon\cdot n=-5\cdot n+W\cdot 5\cdot n+b \qquad \Rightarrow\left(s\cdot\varepsilon\cdot I_2+5\cdot I_2-5\cdot W\right)\cdot n=b$

$$P(s)=\det\left(s\cdot\varepsilon\cdot I_2+5\cdot I_2-5\cdot W\right)=\det\begin{bmatrix}s\cdot\varepsilon-5 & 15p\\-5p & s\cdot\varepsilon+10\end{bmatrix}$$

$$=(s\cdot\varepsilon)^2+5\cdot(s\cdot\varepsilon)+\left(75p^2-50\right)$$

Roots: (always assume $\varepsilon>0$.)

$$(s\cdot\varepsilon)_{1,2}=-\frac{5}{2}\pm\frac{5}{2}\cdot\sqrt{9-12p^2}\ .$$

i) For $|p|>\sqrt{3/4},\ (s\cdot\varepsilon)_{1,2}=-\frac{5}{2}\pm\frac{5}{2}\cdot j\cdot\sqrt{12p^2-9}$, $\mathrm{Re}(s_{1,2})<0$, stable.

ii) For $\sqrt{2/3}\le|p|\le\sqrt{3/4},\ (s\cdot\varepsilon)_{1,2}\le 0$, stable.

iii) For $|p|<\sqrt{2/3}$, one of $(s\cdot\varepsilon)_{1,2}>0$, not stable.

There form when $|p|\ge\sqrt{2/3}$, stable.