ENEE434 Spring 2005

Homework 5 Solution: Y.J. 03/15/05

**Problem 1**: (E18.3 , pp.18-40 of the textbook.) replace $2a_1a_2$ with $-2a_1a_2$ in V(a).

$$V(a) = -\frac{1}{2}\left(a_1^2 - 2a_1a_2 + 4a_2^2 + 6a_1 + 10a_2\right)$$

i) Weight matrix and bias vector:

$$V(a) = -\frac{1}{2}\left(a_1^2 - 2a_1a_2 + 4a_2^2 + 6a_1 + 10a_2\right)$$

$$= -\frac{1}{2}\left(a_1^2 - 2a_1a_2 + 4a_2^2\right) - \left(3a_1 + 5a_2\right)$$

$$= -\frac{1}{2}a^T \cdot \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix} \cdot a - \begin{bmatrix} 3 \\ 5 \end{bmatrix}^T \cdot a$$

Therefore: (compared with equation 18.52, pp. 18-14 in text book, high gain case)

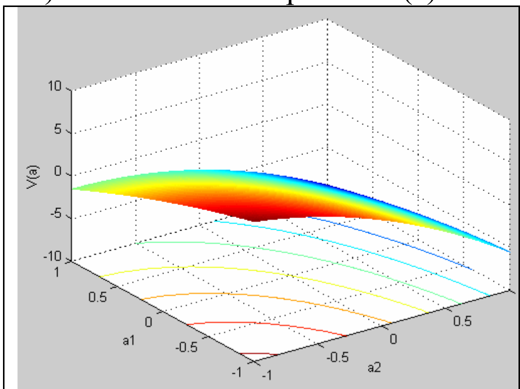
Weight matrix: $W = \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$, and bias vector: $b = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

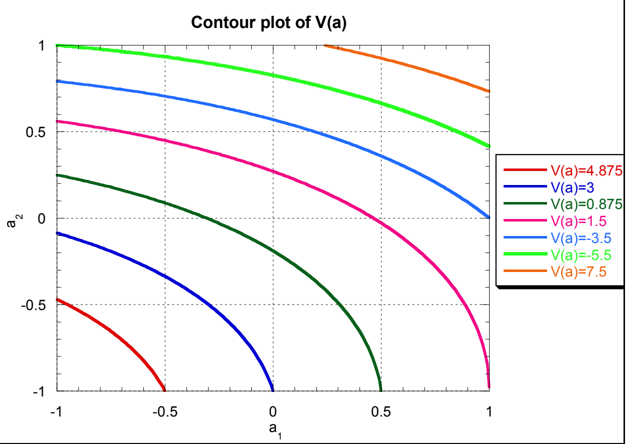
ii) Gradient and Hessian for V(a):

Gradient: $\nabla V(a) = -W \cdot a - b = -\begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix} \cdot a - \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Hessian: $\nabla^2 V(a) = -W = \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix}$

iii) Sketch a contour plot of V(a):




iv) The stationary point(s) for V(a):

$$\left|\nabla^2 V(a) - \lambda I\right| = \lambda^2 - 5\lambda + 3 = 0$$

$\lambda_1 = -0.6972$, and $\lambda_2 = -4.3028$. Since both eigenvalues are negative, V(a) has a single maximum point. To find this maximum: $\nabla V(a) = \begin{bmatrix} -a_1 + a_2 - 3 \\ a_1 - 4a_2 - 5 \end{bmatrix} = 0$, maximum $\left(-\frac{17}{3}, -\frac{8}{3}\right)$ which is out side of the hypercube {$a$:-1<$a_i$<1, i=1, 2}.

Therefore the only stationary point is a=[1 1]$^T$. This point is the only attractor with attraction region of -1<$a_i$<1, i=1, 2.

**Problem 2**:

Design an Hopfield continuous time neural network with activation function a=tanh(n): Finding the weight matrix W and bias vector b to make the Lyaponov function to have only one single minimum at [1 1]$^T$ in the region of -1<$a_i$<1, i=1, 2.

$$V(a) = -\frac{1}{2} \cdot a^T \cdot W \cdot a + \sum_{i=1}^{S} \left\{ \int_0^{a_i} f^{-1}(u) \cdot du \right\} - b^T \cdot a \quad \text{(equation 18.3, pp. 18-5.)}$$

$$\frac{d}{dt} V(a) = \left[ -a^T \cdot W + n^T - b^T \right] \cdot \frac{da}{dt} \quad \text{(equation 18.13, pp. 18-6.)}$$

At equilibrium, dV(a)/dt=0, therefore:

$$-W \cdot a + n - b = 0$$

In this problem, the activation function a=tanh(n);

$$\begin{cases} a_i = \tanh(n_i) \\ n_i = \frac{1}{2} \ln\left(\frac{1+a_i}{1-a_i}\right) \end{cases} \quad i = 1,2$$

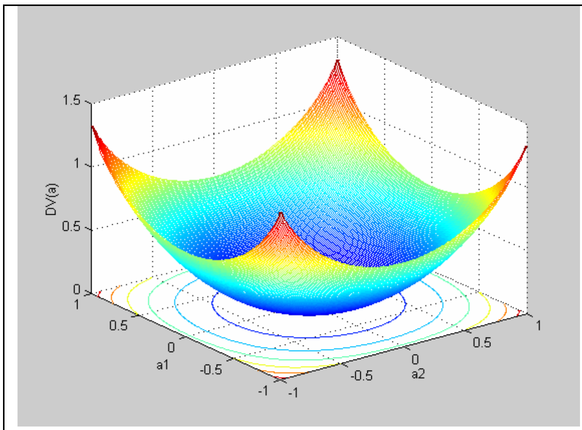
With $a_i$=1, $n_i$=+∞, no solution of W and b to make dV(a)/dt=0. a=[1 1]$^T$ is a boundary minimum. Since the minimum is at the boundary, the solution of this problem is not unique.

$$I(a_i) = \int_0^{a_i} f^{-1}(u) \cdot du = \int_0^{a_i} \frac{1}{2} \ln\left(\frac{1+u}{1-u}\right) \cdot du$$

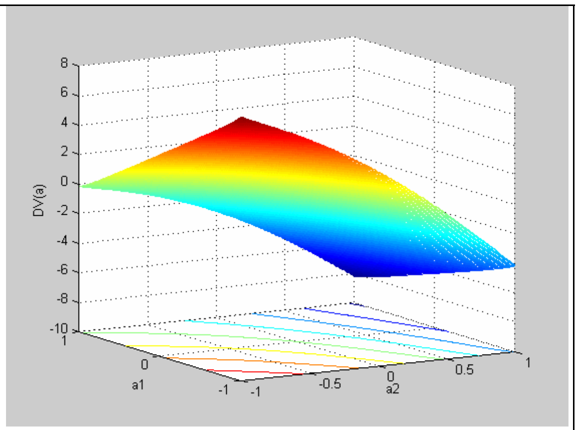
$$= \frac{a_i}{2} \cdot \ln\left(\frac{1+a_i}{1-a_i}\right) - \ln\left\{ \cosh\left[ \frac{1}{2} \ln\left(\frac{1+a_i}{1-a_i}\right) \right] \right\} \qquad i = 1,2$$

$$DV(a) = \sum_{i=1}^{2} \frac{a_i}{2} \cdot \ln\left(\frac{1+a_i}{1-a_i}\right) - \ln\left\{ \cosh\left[ \frac{1}{2} \ln\left(\frac{1+a_i}{1-a_i}\right) \right] \right\}$$

First, let us try if the W and b from problem 1 would give a minimum at [1, 1]$^T$. The plot on the right below shows that [1, 1]$^T$ is the minimum point for V(a). Therefore, the weight W and bias b from problem 1 is one of the solutions.




DV(a)




V(a)

Check with matlab, weight matrix W and bias vector b from problem 1 were used.

```
T=[1;1];
   net=newhop(T);
   net.layers{1}.transferFcn='tanh';
   net.lw{1,1}=[1 -1;-1 4]
   net.b{1,1}=[3;5]

   Ai=T;
   [Y,Pf,Af] = sim(net,1,[],Ai);
   Y
```

$Y=[0.9951, 1.0000]^T$. This is the minimum point.

One solution would be:

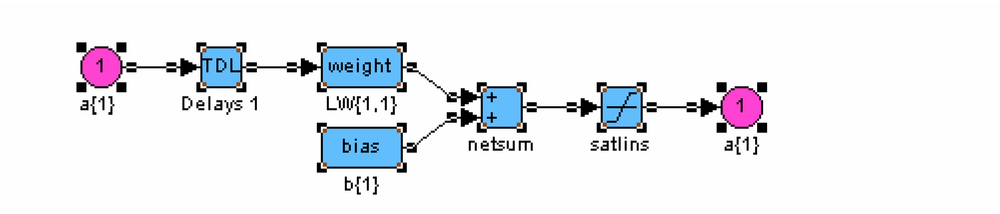
Weight matrix: $W = \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$, and bias vector: $b = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

**Problem 3**:

a) Architectural structures of continuous time and discrete time Hopfiled neural networks.

Continuous time: (Figure 18. 2, pp. 18-5 from Text book,)

Discrete time:




b) Comparison:

| Continuous time | Discrete time |
|---|---|
| Differential Operator on n | 1 delay on a |