ENEE 434 Homework 6 due Th April 14, 2005

1. [30 points]

   An activation function is given by

   $$f(n)=3[-|n+1.5| + 1.5|n+1| - 1.5|n-1| + |n-1.5|]$$

   a) Sketch f(n) for all real n in the interval [-4, +4] labeling important points

   b) Sketch the derivative of f(3n-4) for the same range.

   c) Write f(3n-4) as a weighted sum of shifted satlins functions

   d) Using the result of c) draw a neural network which realizes f(3n-4).

2. [35 points, 20 minutes]

   A digital filter is described by

   $$\frac{y}{u}(z) = \frac{a+5z^{-1}}{1+(1/7)az^{-1}}$$

   a) With a as a parameter, give a neural network realization of this digital filter (including connections, weights, delays, activation functions).

   b) For u(t)=1(t) the unit step function and y(t)=0 for t<1

   give y(t) for t=1, 2, 3, 4 for the three cases of a=3, 7 and a=1/3

   sketch the resulting y(t) in these three cases.

   c) Briefly discuss how you could use this neural network to approximate the output to be f(t), where f(.) is the function in Problem 1 above, when the input is the u(t) of part b) of this problem.

3. [30 points, 20 minutes]

   In the continuous time Hopfield neural network of equations (18.6) & (18.7) has purelin(5n) for all entries of the activation function vector. In this case

   a) Calculate the equilibrium points in terms of generic W, b, ε.

   b)) Discuss the validity of the energy function V(3a-5) as a Lyapunov function, where V(a) is given by equation (18.8).

   c) For the following special case find the equilibrium points and discuss the neural networks' stability with respect to the real parameter p.

$$(1/3)\frac{dn}{dt} = -5n + \begin{bmatrix} 2 & -3p \\ p & -1 \end{bmatrix} a + \begin{bmatrix} +5 \\ -9 \end{bmatrix}, \quad n = \begin{bmatrix} n1 \\ n2 \end{bmatrix}$$

$$a = \begin{bmatrix} \text{purelin}(5n1) \\ \text{purelin}(5n2) \end{bmatrix}$$