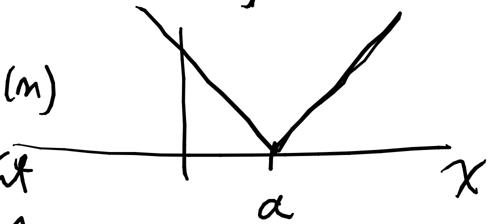
EE 434
03/15/05

$$\mathrm{abs}(x-a) = |x-a|$$

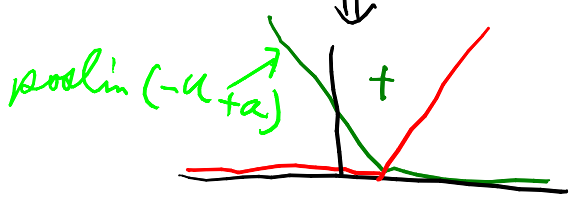


P.2-17

$\mathrm{poslin}(n) = n\,1(n)$

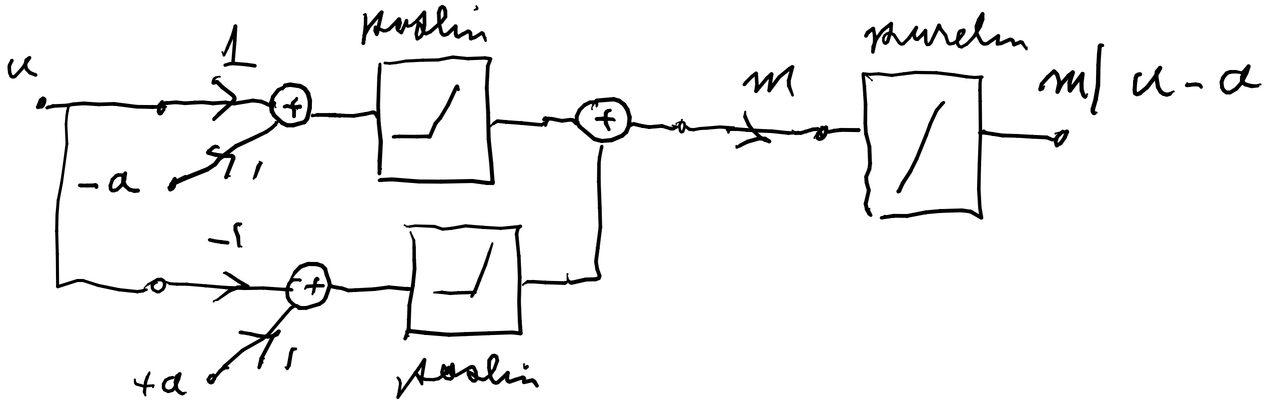
= unit ramp

slope 1



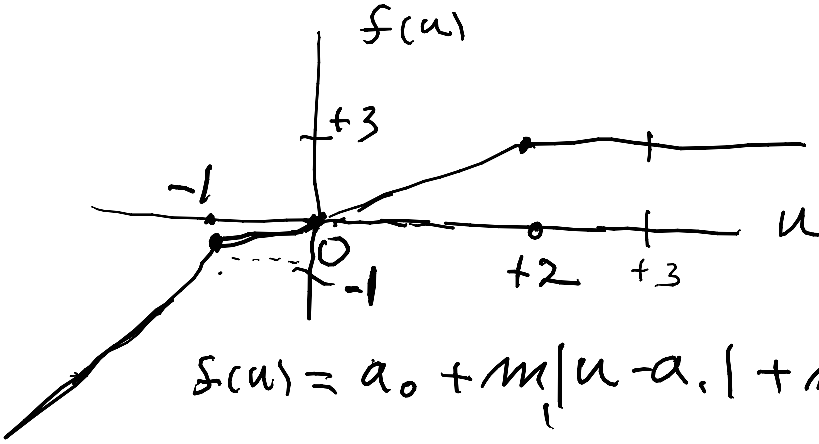
u = input

$$m|u-a|$$
$$= m[\mathrm{poslin}(u-a) + \mathrm{poslin}(-u-(-a))]$$



given a piecewise linear $f(u)$ with a finite number of break points a_i, and continuous then $f(u)$ can be represented as

$$f(u) = a_0 + \sum_{i=1}^{N} m_i |u-a_i|$$



$$f(u) = a_0 + m_1|u-a_1| + m_2|u-a_2| + m_3|u-a_3|$$

$$a_1 = -1, \quad a_2 = 0, \quad a_3 = 2$$

then $f(-1) = a_0 + 0 + m_2|-1-0| + m_3|-1+2|$

$$\overset{\shortparallel}{-1} = a_0 + m_2 + m_3$$

$$0 = f(0) = a_0 + m_1 + 0 + 2m_3$$

$$3 = f(2) = a_0 + 3m_1 + 2m_2 + 0$$

$$3 = f(3) = a_0 + 4m_1 + 3m_2 + m_3$$

$$\Rightarrow \begin{bmatrix} -1 \\ 0 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 3 & 2 & 0 \\ 1 & 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ m_1 \\ m_2 \\ m_3 \end{bmatrix} ; \quad \begin{aligned} A &= Bx \\ x &= B^{-1}A \end{aligned}$$

$$A = B T_1 T_1^{-1} x \quad ; \quad T_1 = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 3 & 2 & 0 \\ 1 & 4 & 3 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{T_1^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & -1 & 2 \\ 1 & 3 & 1 & 0 \\ 1 & 4 & 2 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 1 & 0 & & -1 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}; \quad T_1^{-1} = \begin{bmatrix} 1 & 0 & & +1 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}; \quad T_2 = \begin{bmatrix} 1 & & & -1 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & -1 & 2 \\ 1 & 3 & 1 & 0 \\ 1 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 1 \\ 1 & 3 & 1 & -1 \\ 1 & 4 & 2 & 0 \end{bmatrix}, \quad T_2^{-1} = \begin{bmatrix} 1 & & & +1 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 1 \\ 1 & 3 & 1 & -1 \\ 1 & 4 & 2 & 0 \end{bmatrix} \overset{T_3}{\begin{bmatrix} 1 & & & \\ & 1 & +1 & \\ & & 1 & \\ & & & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 3 & 4 & -1 \\ 1 & 4 & 6 & 0 \end{bmatrix}, \quad T_3^{-1} = \begin{bmatrix} 1 & & & \\ & 1 & -1 & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & +1 \\ 1 & 3 & 4 & -1 \\ 1 & 4 & 6 & 0 \end{bmatrix} \overset{T_4}{\begin{bmatrix} 1 & & & \\ & 1 & & -1 \\ & & 1 & \\ & & & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 3 & 4 & -4 \\ 1 & 4 & 6 & -4 \end{bmatrix}, \quad T_4^{-1} = \begin{bmatrix} 1 & & & \\ & 1 & & +1 \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 3 & 4 & -4 \\ 1 & 4 & 6 & -4 \end{bmatrix} \overset{T_5}{\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & 1 \\ & & & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 \\ 1 & 4 & 6 & -4 \end{bmatrix}, \quad T_5^{-1} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & -1 \\ & & & 1 \end{bmatrix}$$

$$A = (B\, T_1 T_2 T_3 T_4 T_5) \cdot [T_5^{-1} T_4^{-1} T_3^{-1} T_2^{-1} T_1^{-1}]\, x$$

$$\begin{bmatrix} -1 \\ 0 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 \\ 1 & 4 & 6 & -4 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix}; \quad \begin{aligned} x_1' &= -1 \\ x_2' &= -x_1 = 1 \\ 4x_3' &= -3x_2' - x_1' + 3 \\ &= -3 + 1 + 3 = 1; \; x_3' = 1/4 \\ -4x_4' &= -6x_3' - 4x_2' - x_1' + 3 \\ &= -\tfrac{6}{4} - 4 + 1 + 3 = -6/4 \end{aligned}$$

$$x'_4 = \frac{6}{16} \Rightarrow \begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \\ x'_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1/4 \\ 3/8 \end{bmatrix} = T_5^{-1} T_4^{-1} T_3^{-1} T_2^{-1} T_1^{-1} x$$

gives $x = \begin{bmatrix} a_0 \\ m_1 \\ m_2 \\ m_3 \end{bmatrix} = T_1 T_2 T_3 T_4 T_5 \begin{bmatrix} -1 \\ 1 \\ 1/4 \\ 3/8 \end{bmatrix}$

$$= T_1 T_2 T_3 T_4 \begin{bmatrix} -1 \\ 1 \\ 5/8 \\ 3/8 \end{bmatrix} = T_1 T_2 T_3 \begin{bmatrix} -1 \\ 5/8 \\ 5/8 \\ 3/8 \end{bmatrix}$$

$$= T_1 T_2 \begin{bmatrix} -1 \\ 5/4 \\ 5/8 \\ 3/8 \end{bmatrix} = T_1 \begin{bmatrix} -11/8 \\ 5/4 \\ 5/8 \\ 3/8 \end{bmatrix} = \begin{bmatrix} -6/8 \\ 5/4 \\ 5/8 \\ 3/8 \end{bmatrix} = \begin{bmatrix} a_0 \\ m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

$$f(u) = -\frac{6}{8} + \frac{6}{4}|u+1| + \frac{5}{8}|u| + \frac{3}{8}|u-2|$$

shows any continuous piecewise linear function with a finite number of break-points (and everything real valued) can be represented by a neural network & by

$$f(u) = a_0 + \sum_{i=1}^{N} m_i |u - a_i|$$

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a useful activation function is the radial basis one $f(n) = e^{-n^2}$