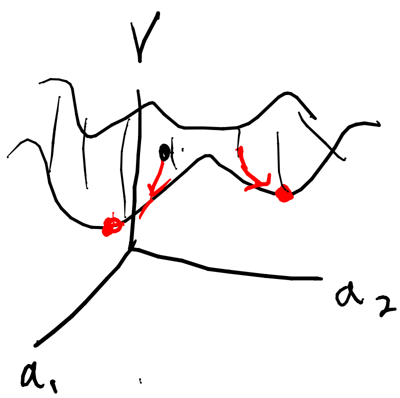
EE 434
03/08/05

Hopfield neural networks
Chapter 18

ODE, motion constrained by an energy function (bounded below)(a negative or 0 time derivative); $V(a) \geq c =$ finite



$$\frac{dV(a)}{dt} \leq 0$$

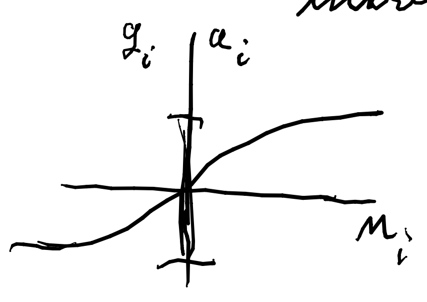
stores information in rest points of ODE

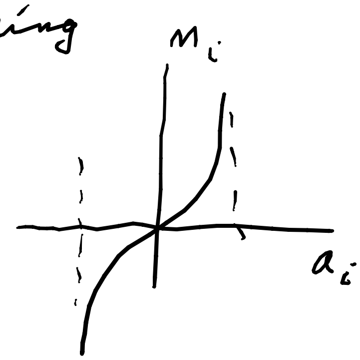
Inputs are initial conditions

$$C\frac{du}{dt} = +Wa - Gu + I$$

$$a = g(u)$$

$g(\cdot) =$ monotone, bounded increasing





a, u, I are S-vectors

W is $S \times S$

C & G = $S \times S$ digonal positive entries

assume $W = W^T$ T = transpose

need $u(0)$

Divide out C by using C^{-1} $\quad \frac{dn}{dt} = +Wa - Gn + I$

$a = g(n), \quad n = g^{-1}(a)$

$\frac{dn}{dt} = \frac{dg^{-1}(a)}{dt} = \frac{dg^{-1}(a)}{da} \cdot \frac{da}{dt}$ here $a_1 = g_1(n_1), \dots, a_s = g_s(n_s)$

hence $\frac{dg^{-1}(a)}{da} = \begin{bmatrix} \frac{dg_1^{-1}(a_1)}{da_1} & & 0 \\ & \frac{dg_2^{-1}(a_2)}{da_2} & \\ 0 & & \ddots \end{bmatrix}$

$(S \times S)$

positive definite $S \times S$ matrix

$\therefore$

$$\left[\frac{dg^{-1}(a)}{da}\right] \cdot \frac{da}{dt} = +Wa - Gg^{-1}(a) + I \quad = -Wa - Gn + I$$

rest points are where $\frac{da}{dt} = 0$; $\frac{dn}{dt} = 0$ also or where

$Wa = -Gn + I$ desire to choose the W and the I so store in certain a's $\Longleftrightarrow$ n's

Energy function $\Longleftrightarrow$ Lyapunov

$$V(a) = -a^T \frac{W}{2} a - a^T I + \sum_{k=1}^{S} G_k \int_0^{a_k} g_k^{-1}(x_k)\, dx_k \qquad (AB)^T = B^T \cdot A^T$$

is bounded below

$$\frac{dV(a)}{dt} = -\frac{da^T}{dt} \frac{W}{2} a - a^T \frac{W}{2} \frac{da}{dt} - \frac{da^T}{dt} \cdot I + \sum_{k=1}^{S} G_k\, g_k^{-1}(a_k) \cdot \frac{da_k}{dt}$$

$$= -a^T \frac{W^T}{2} \frac{da}{dt} - a^T \frac{W}{2} \frac{da}{dt} - I^T \frac{da}{dt} + n^T G^T \frac{da}{dt}$$

note $\left(\frac{dV(a)}{dt}\right)^T = \left(\frac{dV(a)}{dt}\right)$ as it is 1×1 matrix = scalar

$$\frac{dV(a)}{dt} = -\left(a^T\left[\frac{W^T+W}{2}\right] + I^T - n^T G^T\right)\frac{da}{dt}$$

now require $W = W^T$

$$\frac{dV(a)}{dt} = -(Wa + I - Gn)^T \underbrace{\left[\frac{dg(a)}{da}\right]^{-1}}_{P} \underbrace{\left[Wa - Gn + I\right]}_{Y}$$

$$= -Y^T \underset{\text{positive definite}}{[P]} Y \leq 0 \quad \text{\& is 0 only if } \frac{da}{dt} = 0 \iff Y$$

$\therefore$ choose W & I so $Y = 0$

$$\left.\begin{array}{l} Wa_1 - Gn_1 + I = \underline{0} \\ Wa_2 - Gn_2 + I = \underline{0} \end{array}\right\} \quad W(a_1 - a_2) = G(n_1 - n_2)$$

Ex: $S = 2$ choose 3 restpoint vectors n_1, n_2, n_3

choose $g_i(n_i) = \tanh n_i = a_i$

choose $n_1 = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$, $n_2 = \begin{bmatrix} 1/2 \\ -3/2 \end{bmatrix}$; $n_3 = \begin{bmatrix} 1/3 \\ -1/2 \end{bmatrix}$

$n_1 - n_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad n_2 - n_3 = \begin{bmatrix} 1/6 \\ -1 \end{bmatrix}$

$a_1 = \begin{bmatrix} \tanh(1/2) \\ \tanh(-1/2) \end{bmatrix}$, $a_2 = \begin{bmatrix} \tanh(1/2) \\ \tanh(-3/2) \end{bmatrix}$, $a_3 = \begin{bmatrix} \tanh(1/3) \\ \tanh(-1/2) \end{bmatrix}$

$$W\left[a_1 - a_2 \;\vdots\; a_2 - a_3\right] = G\left[n_1 - n_2 \;\vdots\; n_2 - n_3\right]$$