ENEE 302 Spring 2005
Midterm solutions by Y. Z.

1.

a). For PMOS. $|V_{GS}|_p = V_{dd} - V_o$, $|V_{DS}|_p = V_{dd} - V_o$.

(13) For NMOS. $V_{GSn} = V_o$, $V_{DS_n} = V_o$

$I_{DP} = I_{DN} \Rightarrow$

$$\frac{1}{2} kP_p \cdot \frac{W_{P_0}}{L_P} \cdot (|V_{GS}|_p - |V_{tp}|)^2 =$$

$$\frac{1}{2} kP_n \cdot \frac{W_n}{L_n} (V_{GSn} - V_{tn})^2$$

$$\Rightarrow W_{P_0} = \frac{kP_n}{kP_p} \cdot \left[\frac{(|V_{GS}|_p - |V_{tp}|)^2}{(V_{GSn} - V_{tn})^2}\right]^{-1} \cdot W_n$$

$$= 2 \cdot \left[\frac{(V_{dd} - V_o - |V_{tp}|)^2}{(V_o - V_{tn})^2}\right]^{-1} \cdot W_n$$

$$= 2 \cdot \left[\frac{(3 - 1.5 - 1)^2}{(1.5 - 1)^2}\right]^{-1} \cdot W_n$$

$$= 2W_n$$

$$= 20\,u$$

b). With $\lambda_n = 0.01\ V^{-1}$, $|\lambda_p| = 0.02V^{-1}$

(12) $I_{DP} = I_{DN} \Rightarrow$

$$\frac{1}{2} kP_p \cdot \frac{W_{Pn}}{L_P} (|V_{GS}|_p - |V_{tp}|)^2 (1 + |\lambda_p| \cdot |V_{DS}|_p)$$

$$= \frac{1}{2} kP_n \frac{W_n}{L_n} (V_{GSn} - V_{tn})^2 (1 + \lambda_n V_{DSN})$$

$$\Rightarrow W_{pn} = \frac{KP_n}{KP_p} \frac{(V_{GSn} - V_{tn})^2}{(|V_{GS}|_p - |V_{tp}|)^2} \cdot \frac{(1+\lambda_n V_{DSn})}{(1+|\lambda_p| |V_{DS}|_p)}$$

$$= W_{p0} \cdot \frac{(1+\lambda_n V_{DSn})}{(1+|\lambda_p| \cdot |V_{DSp}|)}$$

$$= W_{p0} \cdot \frac{(1+0.01 \times 1.5)}{(1+0.02 \times 1.5)}$$

$$= W_{p0} \cdot 0.9854$$

$$= 19.7\ u$$

c). In part b, when we consider the
(5) Early effect, the drain voltage has effect on the drain current in the mos saturation region. This will make the calculation of W_p slightly different from the case where Early effect is ignored.

2. a). At $t=0$,

(5) $V_a = 5V$, $V_b = 0V$.

$V_a > V_b$. For the pass transistor (Nmos)

a is the drain. b is the source.

b). The differential equation for $V_a(t)$ is

(15) $$i_{DN} \cdot dt = -C_{in}\, dV_a(t)$$

For $V_b(t)$,

$$i_{DN} \cdot dt = C_{out} \cdot dV_b(t)$$

In saturation region

$$i_{DN} = \frac{1}{2} KP \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} KP \frac{W}{L} \cdot (5 - V_b - V_t)^2$$

c). There are two possibilities for the

(15) final states of V_a and V_b when charging and discharging are complete.

Possibility 1: $V_a = V_b$ and $V_b < 4V$

Possibility 2: $V_b = 4V$. and $V_a \geq 4V$

For both possibilities, V_b cannot exceed 4V. Because when $V_b \geq 4V$ ($V_{dd} - V_{t0} = 4V$), the nmos pass transistor is shut down due to $V_{GS} < V_{t0}$.

When the charging of C_{out} and discharging of C_{in} begin, C_{in} keeps discharging, so V_a keeps decreasing. The charges that were initially stored on C_{in} move towards C_{out}, so V_b keeps increasing. If before V_b rises to 4V, V_a has dropped to the same magnitude of V_b, then no charge will move any more since $V_a = V_b$. This is one possibility of final states. But if V_b rises to 4V and V_a is still larger than V_b, case will be different. Charges stop moving because when V_b rises to 4V, the pass tranistor is shut down. So V_b will be fixed at 4V, and V_a stays at its own level ($V_a > 4V$). This case is possibility No. 2.

Which one is the final state in this problem? This depends on the capacitance values of C_{in} and C_{out}, because these values decide the speed of voltage increase or drop.

Let's start from possibility one, and see if this can happen or not.

From the fact that the total charges on these two capacitors are constant all the time, we have the equation

$$\underbrace{C_{in} \cdot V_{a}(t=0)}_{\text{charges at the beginning}} = \underbrace{C_{in} \cdot V_{a}(t_{final}) + C_{out} \cdot V_{b}(t_{final})}_{\text{charges at the end.}}$$

If $V_a(t_{final}) = V_b(t_{final})$, we can calculate

$$V_{a,b}(t_{final}) = \frac{C_{in}}{C_{in} + C_{out}} \cdot V_a(t=0) = \frac{20pF}{20pF + 2pF} \times 5$$

$$= 4.54\,V > 4V$$

V_b (t_{final}) cannot be larger than 4 V from our previous analysis. So possibility one is ruled out.

Let's consider the other possibity, where $V_b(t_{final}) = 4V$.

From constant charge, we have

$$C_{in} \cdot V_a(t=0) = C_{in} \cdot V_a(t_{final}) + C_{out} \cdot 4V$$

$$\Rightarrow V_a(t_{final}) = -\frac{C_{out}}{C_{in}} \cdot 4V + V_a(t=0)$$

$$= 5 - \frac{1}{10} \times 4V$$

$$= 4.6\ V$$

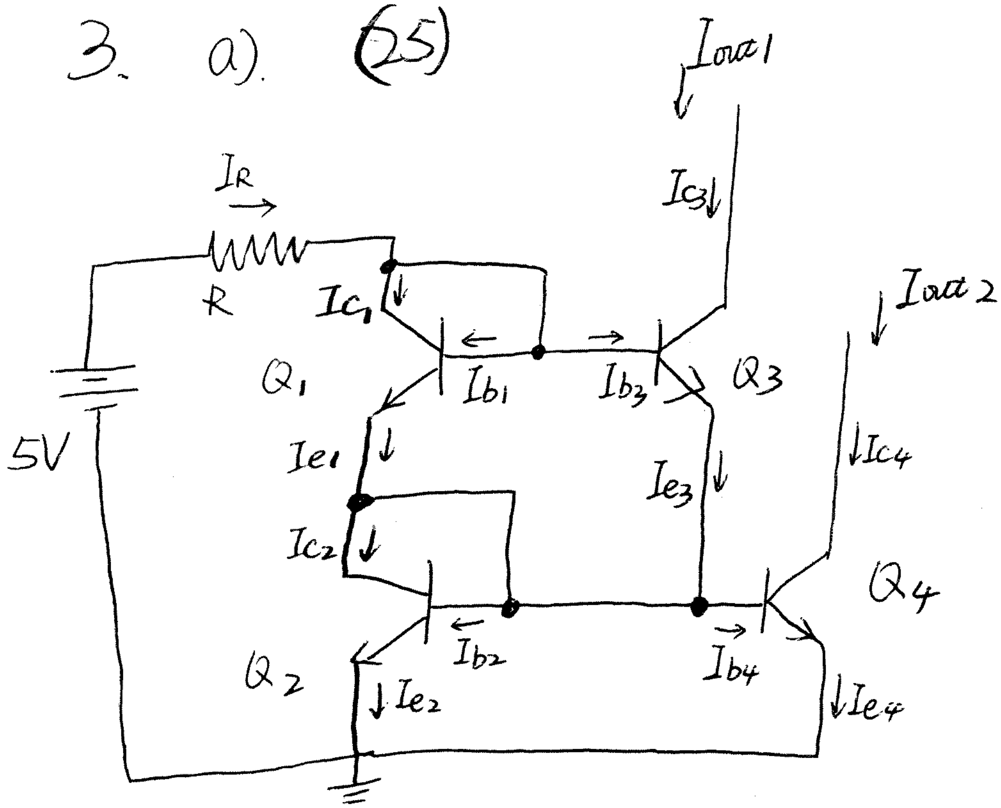
This satisfies $V_a > 4V$.
So this possibity is the real final state of the circuit.

$$V_a(final) = 4.6V$$

$$V_b(final) = 4\ V$$

A is the drain, B is the source.

3. a). (25)



To find R that give $I_{out2} = I_{C4} = 0.2\ mA$, we need to know the current that flows through R (I_R) and the voltage drop accross R (V_R).

To find I_R, see below derivations:

$$I_R = I_{C1} + I_{b1} + I_{b3}$$

$$= I_{C1} + 2I_{b1} \qquad (I_{b1} = I_{b3})$$

$$= \left(1 + \frac{2}{\beta}\right) I_{C1} \qquad (I_C = \beta I_b)$$

$$= (1+\frac{2}{\beta}) \cdot \frac{\beta}{\beta+1} I_{e1} \qquad (I_c = \frac{\beta}{\beta+1} I_e)$$

to find I_{e1}, observe for Q_2 and Q_4, we have

$$I_{e1} + I_{e3} = I_{c2} + I_{b2} + I_{b4},$$

$$\Rightarrow \quad 2I_{e1} = I_{c2} + I_{b2} + I_{b4} \qquad (I_{e1} = I_{e3})$$

$$\Rightarrow \quad I_{e1} = \frac{1}{2}(I_{c2} + I_{b2} + I_{b4})$$

$$= \frac{1}{2}(1+\frac{2}{\beta}) I_{c2} \qquad (I_{b2} = I_{b4}, \; I_c = \beta I_b)$$

$$= \frac{1}{2}(1+\frac{2}{\beta}) I_{c4}$$

$$= \frac{1}{2}(1+\frac{2}{\beta}) I_{out2}$$

so $$I_R = (1+\frac{2}{\beta}) \frac{\beta}{\beta+1} I_{e1}$$

$$= \frac{1}{2}(1+\frac{2}{\beta})^2 \frac{\beta}{\beta+1} I_{out2}$$

$$= \frac{1}{2} \times (1+\frac{2}{5})^2 \cdot \frac{5}{5+1} \times 0.2\,mA$$

$$= 0.163\ mA$$

Next step is to find the two diode drop V_{BE1} and V_{BE2}, so we can know the voltage drop across R.

to find V_{BE1}, we need to know I_{e1}.

From Previous calculation.

$$I_{e1} = \frac{1}{2}\left(1+\frac{2}{\beta}\right) I_{out2} = 0.14 \text{ mA}$$

From I_E formula

$$I_e = I_s' \cdot \exp\left(\frac{V_{BE}}{V_T}\right) \quad \text{and } V_{BE} = 0.7V$$

I_s' ↑ constant

$$\Rightarrow I_e = 1mA.$$

$$\cancel{I_{e1}} \quad V_{BE1} = V_T \log \frac{I_{e1}}{1mA} + 0.7V$$

$$= 0.026 \log \frac{0.14 mA}{1.0 mA} + 0.7V$$

$$= 0.649 \quad V$$

to find V_{BE2}, we need to know I_{e2}.

$$I_{e2} = \frac{\beta+1}{\beta} I_{C2} = \frac{\beta+1}{\beta} I_{out2} = 0.24mA$$

$$V_{BE2} = V_T \log \frac{0.24mA}{1 mA} + 0.7V = 0.663 V$$

So, voltage drop on R is

$$V_R = 5 - V_{BE1} - V_{BE2} = 5 - 0.649 - 0.663$$

$$= 3.688 V$$

so $R = \frac{V_R}{I_R} = \frac{3.688\ V}{0.163\ mA} = 22.62\ k\Omega$

b). $I_{out1} = I_{C3} = I_{C1} = \frac{\beta}{\beta+1} I_{e1}$

(5)

$= \frac{5}{6} \times 0.14\ mA$

$= 0.117\ mA$