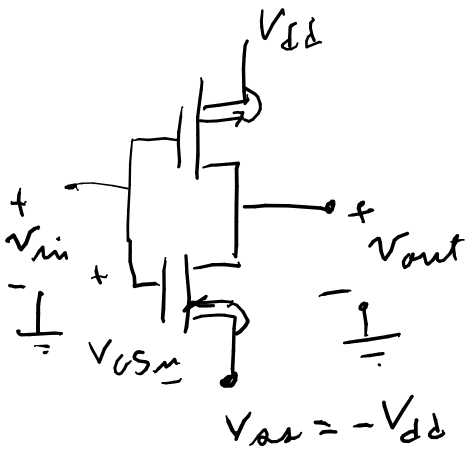
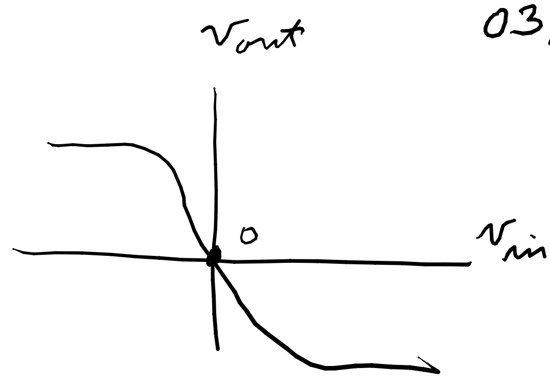
EE 302
03/08/05





$v_{in} = v_{out} = 0$ at transition

$V_{GS_n} = v_{in} - V_{ss}$, $V_{DS_n} = v_{out} - V_{ss} = v_{out} + V_{dd}$

$= v_{in} + V_{dd}$

$V_{GS_n} - VTO_n = 0 + V_{dd} - VTO_n < V_{DS_n} = 0 + V_{dd}$

$\Rightarrow$ NMOS is in saturation

$I_{D_n} = \frac{KP_n}{2}\left(\frac{W}{L}\right)_n (V_{GS_n} - VTO_n)^2 (1 + \lambda_n V_{DS_n})$

$= \beta_n (V_{dd} - VTO_n)^2 (1 + \lambda_n V_{DD})$; at transition midpoint

$\beta_n = \frac{KP_n}{2}\left(\frac{W}{L}\right)_n$

PMOS also in saturation

$V_{SG} = V_{dd} - v_{in}$ $V_{SD} = V_{dd} - v_{out}$

$-I_{D_p} = \beta_p (V_{dd} - |VTO_p|)^2 (1 + \lambda_p V_{dd})$

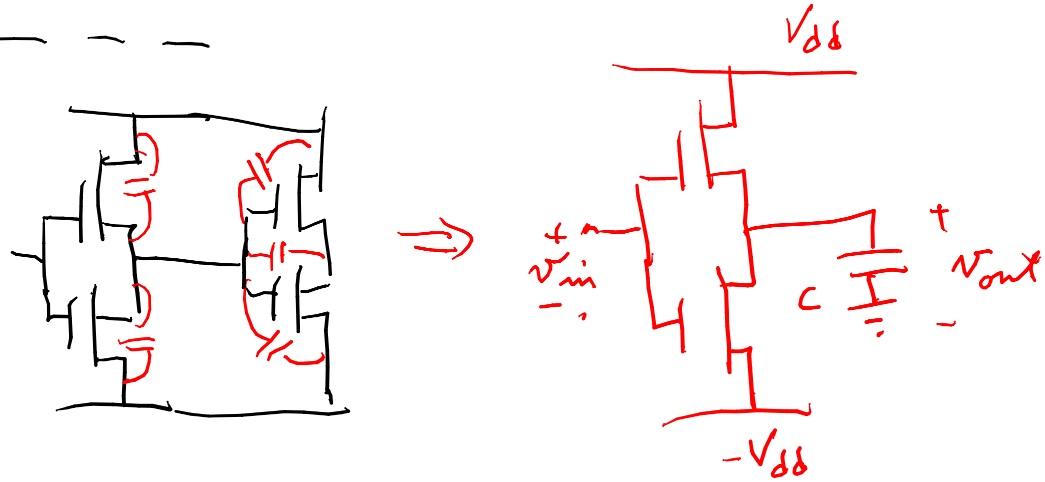
KCL: $I_{D_n} = -I_{D_p}$ gives an equation to design the inverter to transition at $v_{in} = 0$

$$\beta_n (V_{dd} - VTO_n)^2 (1 + \lambda_n V_{DD}) = \beta_p (V_{dd} - |VTO_p|)^2 (1 + \lambda_p V_{dd})$$

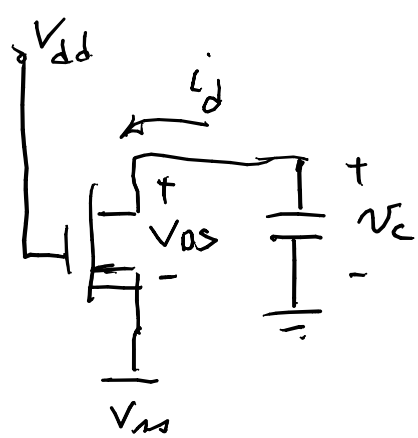
$$\beta_p = \beta_n \left(\left(\frac{V_{dd} - VTO_n}{V_{dd} - |VTO_p|} \right)^2 \left(\frac{1 + \lambda_n V_{dd}}{1 + \lambda_p V_{dd}} \right) \right)$$

if fix $\left(\frac{W}{L}\right)_n$ then this will give $\left(\frac{W}{L}\right)_p$

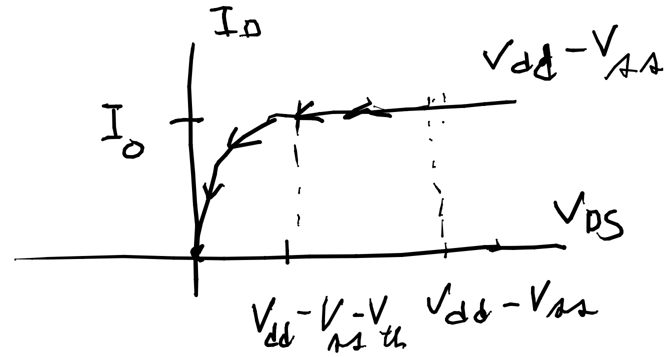
$$\left(\frac{W}{L}\right)_p = \left(\frac{KP_n}{KP_p}\right) \cdot \left(\frac{W}{L}\right)_n \left(\left(\frac{V_{dd} - VTO_n}{V_{dd} - |VTO_p|} \right)^2 \left(\frac{1 + \lambda_n V_{dd}}{1 + \lambda_p V_{dd}} \right) \right)$$



at $t = 0$



$v_c(0_+) = V_{dd}$



i_d flows until drops to 0

stay in saturation as long as $V_{DS_n} > V_{dd} - V_{ss} - VTO_n$

$V_{DS_n} = v_C - V_{ss}$ $\qquad\qquad$ $\overset{\shortparallel}{v_C - V_{ss}}$

that is as long as $v_C > V_{dd} - V_{th}$

here $i_{D_n} = \beta(V_{GS} - VTO_n)^2(1 + \lambda V_{DS})$

$$= \beta(V_{DD} - V_{ss} - VTO_n)^2(1 + \lambda[v_C - V_{ss}])$$

$$C\frac{dv_c}{dt} = -i_{D_n} = -\beta(2V_{dd} - VTO_n)^2(1 + \lambda v_C + \lambda V_{dd})$$

while in saturation as long as $v_C > V_{dd} - VTO_n$

to get time to do this

$$\frac{-C}{\beta(2V_{dd} - VTO_n)^2} \cdot \frac{dv_c}{(1 + \lambda V_{dd} + \lambda v_c)} = 1\,dt$$

$$\frac{-C}{\beta(2V_{dd} - VTO_n)^2} \int_{V_{dd}}^{V_{dd} - VTO_n} \frac{dv_c}{((1 + \lambda V_{dd}) + \lambda v_c)} = \int_0^{t_{fs}} dt$$

$$\underset{\substack{\shortparallel \\ \text{time to} \\ \text{get to ohmic}}}{t_{fs}} = \frac{C}{\beta(2V_{dd} - VTO_n)^2} \frac{1}{1 + \lambda V_{dd}} \cdot \underbrace{\int_{V_{dd} - VTO_n}^{V_{dd}} \frac{dx}{\left(1 + \frac{\lambda}{1 + \lambda V_{dd}} x\right)}}_{\frac{1 + \lambda V_{dd}}{\lambda} \cdot \ln\left(1 + \frac{\lambda}{1 + \lambda V_{dd}} x\right)\Big|_{x = V_{dd} - V_{th}}^{V_{dd}}} \qquad x = v_c$$

$$\frac{d(\ln(1 + ax))}{dx} = \frac{1}{1 + ax} \times a$$

gives

$$t_{fd} = \frac{C}{\beta(2V_{dd}-VTO_n)^2}\cdot\cancel{\frac{1}{1+\lambda V_{dd}}}\times\cancel{\frac{1+\lambda V_{dd}}{\lambda}}\cdot \ln\left(\frac{1+\frac{\lambda}{1+\lambda V_{dd}}V_{dd}}{1+\frac{\lambda}{1+\lambda V_{dd}}(V_{dd}-V_{th})}\right)$$

Eq. 4.152 : $\dfrac{C\,V_{th}}{\beta(2V_{dd}-VTO_n)^2}$ which is the above as $\lambda \to 0$

at this time $v_c = V_{dd} - VTO_n$

Now the transistor becomes ohmic

$$i_{d_n} = \beta\left(2(V_{GS}-VTO_n)V_{DS} - V_{DS}^2\right); \quad V_{DS} = v_c - V_{ss} = v_c + V_{dd}$$

$$= -C\frac{dv_c}{dt} = \beta\left(2(2V_{dd}-VTO_n)(v_c+V_{dd}) - (v_c+V_{dd})^2\right)$$

$$\frac{dv_c}{dt} = -\frac{\beta}{C}\left[V_{dd}\left(2(2V_{dd}-VTO_n)-V_{dd}\right) + (2V_{dd}-2VTO_n)v_c - v_c^2\right]$$

$$dt = \frac{C}{\beta}\cdot\frac{dv_c}{v_c^2 - 2(V_{dd}-VTO_n)v_c - V_{dd}\left(2(2V_{dd}-VTO_n)-V_{dd}\right)}$$

$$t_2 = \frac{C}{\beta}\int_{V_{dd}-VTO_n}^{0}\frac{dv_c}{v_c^2-(a)v_c-(b)}$$

b = 0 in book (4.154) as $V_{ss} = 0$

gives the time to fully discharge the capacitor.
(will keep "discharging" [charges in opposite direction] until hit $V_{ss} = -V_{dd}$)

$t_2 + t_{fd}$ = time to change the state of the next gate
note as $C\downarrow$ this time decreases