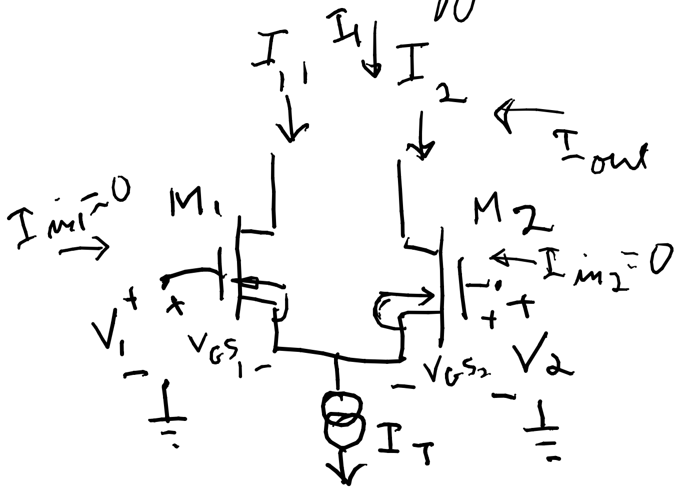
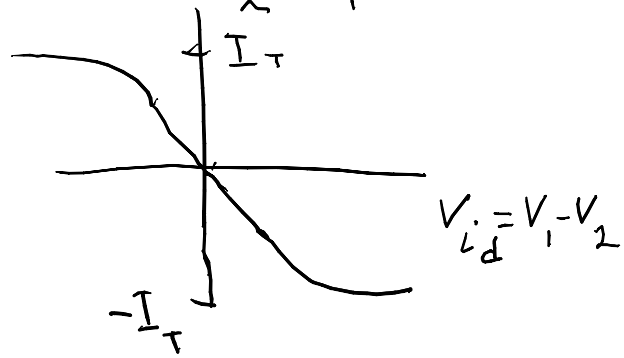
P. 693 MOS differential pair

EE302
02/22/05



steer I_T $\quad I_2 - I_1 = I_{out}$



$$I_T = I_1 + I_2$$
$$I_{out} = I_2 - I_1 = I_0$$

for saturation

$V_{DS} \geq V_{GS} - V_{th}$, $V_{th} = VTO$

when $V_{BS} = 0$

$$I_D = \underbrace{\frac{KP}{2} \times \frac{W}{L}}_{\beta} (V_{GS} - V_{th})^2 \quad \underbrace{(1 + \lambda V_{DS})}_{\text{ignore here}}$$

assume $\beta_1 = \beta_2 = \beta$; $\quad I_1 = \beta(V_{GS_1} - VTO)^2$ $\qquad V_{GS_1} - V_{GS_2} = V_1 - V_2$

$$I_2 = \beta(V_{GS_2} - VTO)^2$$

Let $x_1^2 = \beta(V_{GS_1} - VTO)^2$, $x_2^2 = \beta(V_{GS_2} - VTO)^2$

$x_1 = +\sqrt{\beta}(V_{GS_1} - VTO)$, $x_2 = +\sqrt{\beta}(V_{GS_2} - VTO)$ use + sign as assume turned on M_1, M_2

$$\left.\begin{aligned} I_T &= x_1^2 + x_2^2 \\ I_0 &= x_2^2 - x_1^2 \end{aligned}\right\} \begin{aligned} 2x_2^2 &= I_T + I_0 \\ 2x_1^2 &= I_T - I_0 \end{aligned}$$

$$x_1 - x_2 = \sqrt{\beta}(V_{GS_1} - V_{GS}) = \sqrt{\beta}\, V_{id}$$

$$x_1 = +\frac{1}{\sqrt{2}}(I_T - I_0)^{1/2} \; ; \; x_2 = \frac{1}{\sqrt{2}}(I_T + I_0)^{1/2}$$

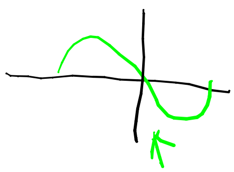
$$\frac{1}{\sqrt{2}}(I_T - I_0)^{1/2} - \frac{1}{\sqrt{2}}(I_T + I_0)^{1/2} = \sqrt{\beta}\, V_{id}$$

$$(I_T - \cancel{I_0}) + (I_T + \cancel{I_0}) - 2\left((I_T - I_0)\times (I + I_0)\right)^{1/2} = 2\beta V_{id}^2$$

$$\cancel{2} I_T - \cancel{2}(I_T^2 - I_0^2)^{1/2} = \cancel{2}\beta V_{id}^2 \Rightarrow I_T - \beta V_{id}^2 = (I_T^2 - I_0^2)^{1/2}$$

square

$$I_T^2 - I_0^2 = (I_T - \beta V_{id}^2)^2 \Rightarrow I_0^2 = \cancel{I_T^2} - (\cancel{I_T^2} - 2\beta I_T V_{id}^2 + \beta^2 V_{id}^4)$$



$$= 2\beta I_T V_{id}^2 \left(1 - \frac{\beta V_{id}^2}{2I_T}\right)$$

$\therefore$

$$I_0 = -\sqrt{2\beta I_T}\cdot V_{id}\sqrt{1 - \frac{\beta V_{id}^2}{2I_T}}$$

need − sign here as chose $I_0 < 0$ for $V_{id} > 0$

look for max of I_0 vs. V_{id}

$$\frac{dI_0}{dV_{id}} = -\sqrt{2\beta I_T}\sqrt{1 - \frac{\beta V_{id}^2}{2I_T}} - \sqrt{2\beta I_T}\, V_{id}\cdot \frac{1}{2}\,\frac{1\times\left(-\frac{2\beta V_{id}}{2I_T}\right)}{\sqrt{1 - \frac{\beta V_{id}^2}{2I_T}}}$$

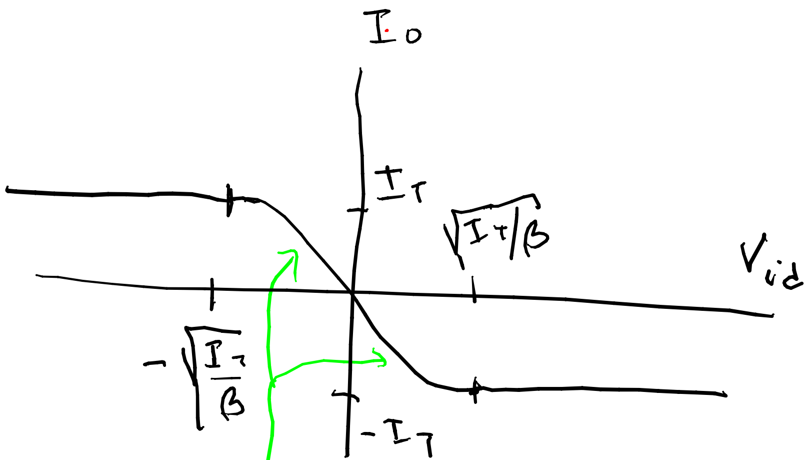
$= 0$

$$\Rightarrow \cancel{\sqrt{2\beta I_T}}\left(1 - \frac{\beta V_{id}^2}{2I_T}\right) = \cancel{\sqrt{2\beta I_T}}\cdot V_{id}\cdot\frac{\beta V_{id}}{2I_T}$$

$$1 = 2\left(\frac{\beta V_{id}^2}{2I_T}\right) \Rightarrow V_{id} = \pm\sqrt{\frac{I_T}{\beta}}$$

for max I_0

$$I_{0_{max}} = I_0\Big|_{V_{id\,max}} = \sqrt{2\cancel{\beta} I_T}\cdot\sqrt{\frac{I_T}{\cancel{\beta}}}\sqrt{1 - \frac{\cancel{\beta} I_T}{2I_T\cancel{\beta}}} = I_T$$



$$I_o = \sqrt{2\beta I_T}\, V_{id}\sqrt{1-\frac{\beta V_{id}^2}{2I_T}}$$

$$I_o = \begin{cases} I_T & V_{id} \le -\sqrt{\frac{I_T}{\beta}} \\ \sqrt{2\beta I_T}\, V_{id}\sqrt{1-\frac{\beta V_{id}^2}{2I_T}} & -\sqrt{\frac{I_T}{\beta}} \le V_{id} \le \sqrt{\frac{I_T}{\beta}} \\ -I_T & \sqrt{\frac{I_T}{\beta}} \le V_{id} \end{cases}$$

If $KP = 7.8 \times 10^{-5}$

$\beta = \frac{KP}{2} \times \frac{W}{L} = 3.9 \times 10^{-5} \cdot \frac{W}{L}$; choose $W = L = 10\mu = 10u$

choose I_T : $I_T = 100\,\mu A$; $\frac{I_T}{\beta} = \cancel{\frac{3.9 \times 10^{-5}}{10^2 \times 10^{-6}}} = \cancel{\frac{3.9}{10}} = \frac{10}{3.9}$

$\sqrt{\frac{I_T}{\beta}} = \cancel{0.62} = 1.6$

$\sqrt{2\beta I_T} = 8.83 \times 10^{-5}$
$= 88.3 \times 10^{-6}$

$2\beta I_T = 2 \times 3.9 \times 10^{-5} \times 10^{-4}$
$= 7.8 \times 10^{-9}$
$= 78 \times 10^{-10}$

$$g_m = \left.\frac{\partial I_o}{\partial V_{id}}\right|_{V_{id}=0} = 88.3 \times 10^{-6}$$