1. tkhm

a) $a_3 = f_3(W_{31} f_1(W_{11}x_1 + W_{12}x_2) + W_{32} f_2(W_{21}x_1 + W_{22}x_2))$

$= W_{31} f_1(W_{11}x_1 + W_{12}x_2) + W_{32} f_2(W_{21}x_1 + W_{22}x_2)$ as f_3 = purelin

b) The restrictions depend upon the classes of f_1 & f_2 allowed. If these are such, as common, that $f_1(0) = f_2(0) = 0$ then $x_1 = x_2 = 0$ implies $a_3 = 0$.

c) For $W_{ij} = 1$ & $f_1 = f_2 = \tanh$ then

$a_3(x) = 2\tanh(x_1 + x_2) = 2\tanh(3/2)$ for $x = \begin{bmatrix}1\\ 1/2\end{bmatrix}$

$= 1.8103\ldots$

$y(x) = \frac{1}{3}(1)^3 + (1)\cdot(1/2) = \frac{1}{3} + \frac{1}{2} = \frac{5}{6} = 0.8333\ldots$

output error $= y(x) - a_3(x) = -0.977\ldots \quad = (t-a)$ for $c_2)$

c₂) $s^M = -2\dot{F}^M(n^M)(t-a) \qquad f_3(x) = x$

$= -2\dot{f}_3(n_3)(y - a_3) \qquad \dot{f}_3(x) = 1$

$\therefore s^M = s^3 = -2\cdot(y - a_3) = +2\times 0.977 = +1.954$

$s^{M-1} = \dot{F}^{M-1}(n^{M-1})(W^{M-1+1})^T s^{M-1+1}$

$\Rightarrow s^2 = \begin{bmatrix}\tanh'(n_1) & 0\\ 0 & \tanh'(n_2)\end{bmatrix}\begin{bmatrix}W_{31}\\ W_{32}\end{bmatrix} s^3$

if $y = \tanh x$
$\tanh' x = (1+y)(1-y)$
$= 1 - y^2$

$\therefore$ need n_1 & n_2: $n_1 = W_{11}x_1 + W_{12}x_2 = 3/2$; $n_2 = W_{21}x_1 + W_{22}x_2 = 3/2$

$\tanh'(n_i) = (1 - \tanh(n_i)) = 1 - \tanh^2(3/2) = 0.1807$

or $s^2 = (0.1807)(+1.954)\begin{bmatrix}1\\1\end{bmatrix} = +0.3531\begin{bmatrix}1\\1\end{bmatrix}$

c₃) $\Delta W_{ij}^m = -\alpha s^m (a^{m-1})^T \Rightarrow [\Delta W_{31}\ \ \Delta W_{32}] = -\alpha s^3 [f_1(n_1),\ f_2(n_2)]$

$= -\alpha(+1.954)[\tanh(3/2),\ \tanh(3/2)]$

$= -\alpha(1.7687)[1,\ 1]$

$\Rightarrow \begin{bmatrix}\Delta W_{11} & \Delta W_{12}\\ \Delta W_{21} & \Delta W_{22}\end{bmatrix} = -\alpha s^2 [x_1,\ x_2] = -\alpha(+0.3531)\begin{bmatrix}1\\1\end{bmatrix}[1,\ 1/2]$

$= -\alpha(0.3531)\begin{bmatrix}1 & 1/2\\ 1 & 1/2\end{bmatrix}$

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c_4) $\underline{W}_{ij} = W_{ij} + \Delta W_{ij}$ gives the new weights

$\underline{W}_{11} = 1 - 0.13531\alpha$, $\underline{W}_{12} = 1 - 0.1766\alpha$
$\underline{W}_{21} = 1 - 0.13531\alpha$, $\underline{W}_{22} = 1 - 0.1766\alpha$ } new weights
$\underline{W}_{31} = 1 - 1.17687\alpha$, $\underline{W}_{32} = 1 - 1.17687\alpha$

new values:

$n_1 = \underline{W}_{11} x_1 + \underline{W}_{12} x_2 = (1 - 0.13531\alpha) + (\frac{1}{2} - 0.0883\alpha)$
$= 3/2 + \alpha(0.4414)$

$n_2 = n_1$

$\tanh(n_1) = \tanh(n_2) = \tanh(\frac{3}{2} - \alpha(0.4414))$

$a_3 = W_{31} \tanh(n_1) + W_{32} \tanh(n_2) = 2(1 - 1.17687\alpha) \tanh(n_1)$
$= (2 + 3.5374\alpha) \tanh(1.5 - (0.4414)\alpha)$

Error; recall $y(x) = 5/6$

$y - a_3 = \frac{5}{6} - (2 - 3.5374\alpha) \tanh(1.5 - (0.4414\alpha))$

If $\alpha = 0.1$; $\tanh(1.45586) = 0.8968$

$y - a_3 = (0.8333\ldots - (1.64626) \times 0.8968$
$= (0.833 - 1.4764)$
$= -0.6430$

Note the error improved from -0.977 to -0.643