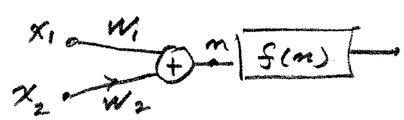
RWN
05/17/04

1 class a) $f(s) = \dfrac{1}{1+\exp(-s)}$ is an analytic extension of the real valued function $\dfrac{1}{1+e^{-\sigma}}$ of the real valued variable σ. Since $\text{logsig}(\sigma) = \dfrac{1}{1+e^{-\sigma}}$ is a useful activation function, its analytic continuation should be useful for complex NNETs.

b) $$f(j\omega) = \frac{1}{1+e^{-j\omega}} = \frac{1}{1+\cos\omega - j\sin\omega}$$

at $\omega = \pi$ we have $\cos\omega = -1$, $\sin\omega = 0 \Rightarrow f(j\pi) \to \infty$ that is, $f(s)$ has a pole at $s = j\pi$ and due to periodicity at $s = j(2k+1)\pi$. So this activation function is not bounded in which case back-propagation can fail!

c) 

$$y = \frac{1}{1+e^{-[w_1x_1+w_2x_2]}}$$

$$y = \frac{1}{(1+e^{-[w_1x_1+w_2x_2]})} \times \frac{(1+e^{-[w_1^*x_1+w_2^*x_2]})}{(1+e^{-[w_1^*x_1+w_2^*x_2]})}, \quad * = \text{complex conjugate}$$

$$= \frac{(1+\operatorname{Re} e^{-[w_1^*x_1+w_2^*x_2]}) + j\operatorname{Im} e^{-[w_1^*x_1+w_2^*x_2]}}{|1+e^{-[w_1x_1+w_2x_2]}|^2}$$

Write $w_1^*x_1 + w_2^*x_2 = (w_{1r} - jw_{1i})x_1 + (w_{2r} - jw_{2i})x_2$
$= (w_{1r}x_1 + w_{2r}x_2) - j(w_{1i}x_1 + w_{2i}x_2)$

so $\operatorname{Im} e^{-[w_1^*x_1+w_2^*x_2]} = e^{-(w_{1r}x_1+w_{2r}x_2)} \cdot \sin(-[w_{1i}x_1 + w_{2i}x_2])$

For $\operatorname{Im} y \equiv 0$ then $\sin(-[w_{1i}x_1 + w_{2i}x_2]) = 0$

$\Rightarrow w_{1i}x_1 + w_{2i}x_2 = k\pi$ for some integer k.

As this must hold for all real x_1 & x_2 we must have $k = 0$, $\Rightarrow w_{1i} = w_{2i} = 0$

$\Rightarrow$ w_1 & w_2 must be real.

KWN
05/17/04

1 class: a) $a_3(x) = f_3(W_3 \cdot f_2(W_2 \cdot f_1(W_1 \cdot x)))$

b) $a_3(x) = W_3 f_2(W_2 \cdot W_1 x) = W_3 e^{-\frac{(W_2 W_1 x - a)^2}{b}} = W_3 e^{-\frac{\left(x - \frac{a}{W_2 W_1}\right)^2}{b/(W_2 W_1)^2}}$

$\therefore$, we have three independent parameters, W_3, $\frac{a}{W_2 W_1}$, $\frac{b}{(W_2 W_1)^2}$ which can be used to fit points (choosing different $W_2 \cdot W_1$ only rescales a & b; $a' = a/(W_2W_1)$, $b' = b/(W_2W_1)^2$

$\therefore$ at most three points can be exactly fit

choose any 3 points $x_1, x_2, x_3 \in [1,2]$ and solve for W_3, a', b'

$x_1^4 = W_3 e^{-\frac{(x_1-a')^2}{b'}}$, $x_2^4 = W_3 e^{-\frac{(x_2-a')^2}{b'}}$, $x_3^4 = W_3 e^{-\frac{(x_3-a')^2}{b'}}$

Possible solution: $\left(\frac{x_1}{x_2}\right)^4 = e^{-\frac{(x_1-a')^2}{b'}} \Big/ e^{-\frac{(x_2-a')^2}{b'}}$

$$\left(\frac{x_1}{x_3}\right)^4 = e^{-\frac{(x_1-a')^2}{b'}} \Big/ e^{-\frac{(x_3-a')^2}{b'}}$$

α) or $\ln\left(\frac{x_1}{x_2}\right)^4 = \ln\left(e^{-\frac{(x_1-a')^2}{b'}}\right) - \ln\left(e^{-\frac{(x_2-a')^2}{b'}}\right) = -\frac{(x_1-a')^2}{b'} + \frac{(x_2-a')^2}{b'}$

by symmetry
β) $\ln\left(\left(\frac{x_1}{x_3}\right)^4\right) = -\frac{(x_1-a')^2}{b'} + \frac{(x_3-a')^2}{b'}$

from α) $b' = [(x_2-a')^2 - (x_1-a')^2] / \ln\left((x_1/x_2)^4\right)$

into β) $\frac{\ln((x_1/x_3)^4)}{\ln((x_1/x_2)^4)} = \frac{(x_3-a')^2 - (x_1-a')^2}{(x_2-a')^2 - (x_1-a')^2}$

This is linear in a' (the a'^2 terms all cancel) and $\therefore$ has a real solution & from α) b' is real. The sign of a' depends upon $\left(\frac{\ln((x_1/x_3)^4)}{\ln((x_1/x_2)^4)}\right)$, that is on the interleaving of x_1, x_2, x_3; this interleaving can be chosen such that $b' > 0$ & then W_1W_2 chosen the sign of a' so that a and b can be forced positive $\Rightarrow$ we can fit exactly 3 points in $[1,2]$.

b_2) $E^2 = \left[1 - e^{-\frac{(1-a)^2}{b}}\right]^2 + \left[16 - e^{-\frac{(2-a)^2}{b}}\right]^2$

as the maximum $e^{-(2-a)^2/b}$ is 1, the minimum of the 1st term would be 0 & the second term 15^2. This is achieved for $a = 1$, $b \to \infty$ (or $a = 2$, $b \to \infty$) and then $E = 15$.