03/30/04c
RWN
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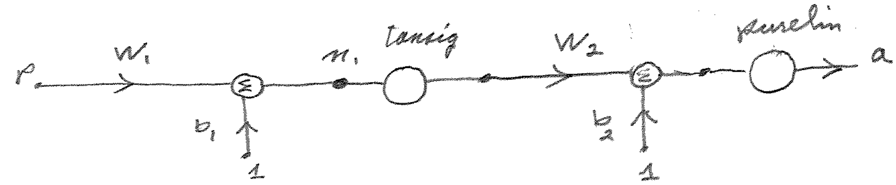
EE434 Midterm Solution

#1. a)
nnetest = newff([-2 3; -2 3; -2 3], [5 2], {'tansig' 'purelin'})
to set the weights & biases
nnetest.IW{1,1} = [1 1 1; 1 1 1; 1 1 1; 1 1 1; 1 1 1]
nnetest.b{1} = [-1; -1; -1; -1; -1]
nnetest.LW{2,1} = [1 1 1 1 1; 1 1 1 1 1]
nnetest.b{2} = [-1; -1]

b)



$$W_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},\quad b_1 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix},\quad W_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix},\quad b_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

let $p_1 = \begin{bmatrix} 1 \\ 1/2 \\ -1 \end{bmatrix}$, $p_2 = \begin{bmatrix} -1 \\ 1/2 \\ 1 \end{bmatrix}$ then both give $W_1 p_i = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$

and $n_1 = -\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$; as $\tanh(1/2) = 0.462$, $a_1 = -\tanh(1/2)\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$$n_2 = W_2 a_1 + b_2 = -\tanh(1/2)\begin{bmatrix} 5 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -3.311\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

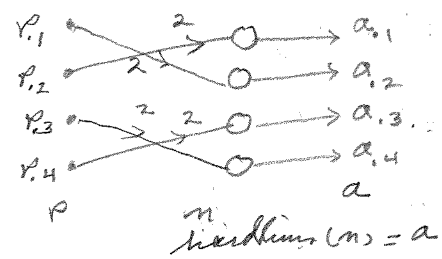
as the output is purelin, in both cases

$$a = -3.311\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

#2, a) $W = PP^T - Q\,1_4$, $Q = 2$, $p_1 = [1\ 1\ 1\ 1]^T$, $p_2 = [1\ 1\ -1\ -1]^T$

$$P = [p_1 \mid p_2] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix};\quad PP^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$$W = PP^T - Q\,1_4 = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$



b$_1$) $n_3 = W p_3 = 2\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = 2\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow a_3 = \text{hardlims}(n_3) = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \neq p_3$

b$_2$) $a_3 = p_3$ should result, as a check

$$W_{new} = [p_1\ p_2\ p_3][p_1\ p_2\ p_3]^T - 3\,1_4 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & -1 \\ -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix};\quad W_{new} p_3 = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & -1 \\ -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = n_3 = a_3 = p_3$$

also $W_{new} p_1 = n_{1new} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = a_{1new} = p_1$; $W_{new} p_2 = n_{2new} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = a_{2new} = p_2$

c) Since W is 4x4 its rank is at most 4 so there can be at most one other, p_4, linearly independent of p_1, p_2, p_3

We want

$$[p_1\ p_2\ p_3][p_1\ p_2\ p_3]^T \cdot p_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 & 1 \\ 1 & 3 & 1 & -1 \\ -1 & 1 & 3 & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} p_{4_1} \\ p_{4_2} \\ p_{4_3} \\ p_{4_4} \end{bmatrix}$$

; this eq. is preserved by adding multiples of rows to rows

4th row × −3 to 1st, × −1 to 2nd, × 1 to 3rd

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -4 & -8 \\ 0 & 4 & 0 & -4 \\ 0 & 0 & 4 & 4 \\ 1 & -1 & 1 & 3 \end{bmatrix} p_4 \Rightarrow$$

−2nd to 1st, 2nd × 1/4 to 4th

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -4 & -4 \\ 0 & 4 & 0 & -4 \\ 0 & 0 & 4 & 4 \\ 1 & 0 & 1 & 2 \end{bmatrix} p_4 \Rightarrow$$

1st to 3rd, 1/4 1st to 4th

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -4 & -4 \\ 0 & 4 & 0 & -4 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} p_4$$

1st & 2nd × 1/4

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} p_4$$

$\therefore p_{4_1} = -p_{4_4}$, $p_{4_3} = -p_{4_4}$, $p_{2_4} = p_{4_4}$

Choose $p_{4_4} = 1 \Rightarrow p_4 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

Then

$$W_4 = [r_1\ r_2\ r_3\ r_4][r_1\ r_2\ r_3\ r_4]^T - 4\,\mathbb{1}_4$$

$$= \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} - 4\mathbb{1}_4 \equiv 0$$

$\therefore$ will always get $w = \underline{0}$ if make W full rank, m, of subtract $m\mathbb{1}_m$ since the r's are orthogonal

d) For $2k-1$ vectors the number of entries is odd so no two can be orthogonal $\Rightarrow$ only one vector can be recognized

For 2^k vectors can choose $2^k - 1$ by pairing in sets of 2, as per above except when add the last one will always get $W_{2^k} = 0$

#3 a) $y = \text{tansig}(x) = \tanh x = \dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}} = \dfrac{e^{2x}-1}{e^{2x}+1}$

$$\frac{d\tanh x}{dx} = \frac{2e^{2x}}{(e^{2x}+1)} - \frac{2e^{2x}}{(e^{2x}+1)^2}(e^{2x}-1) = \frac{2e^{2x}(e^{2x}+1-e^{2x}+1)}{(e^{2x}+1)^2} = \frac{4e^{2x}}{(e^{2x}+1)^2}$$

$\therefore$ we want e^{2x} in terms of y: $(e^{2x}+1)\,y = e^{2x}-1$

$$\Rightarrow \quad e^{2x}(y-1) = -y-1 \Rightarrow e^{2x} = \frac{1+y}{1-y}$$

$$\therefore \quad \frac{d\tanh x}{dx} = \frac{dy}{dx} = \frac{4\left(\frac{1+y}{1-y}\right)}{\left(\frac{1+y}{1-y}+1\right)^2} = \frac{4(1+y)(1-y)}{(2)^2} = (1-y^2)$$

b) 

$$W^1(1) = W^1(0) - \alpha\, s^1(0)\, a^0(0)^T \; ; \quad s(0) = s^M(0) = -2\hat{F}^M(n^M)(t-a)\Big|_{k=0}$$

$$\hat{F}^M(n) = \begin{bmatrix} \frac{d}{dn_1}\text{logsig}(n_1) & 0 \\ 0 & \frac{d\,\text{logsig}(n_2)}{dn_2} \end{bmatrix} = \begin{bmatrix} (1-a_1)a_1 & 0 \\ 0 & (1-a_2)a_2 \end{bmatrix}$$

Now $a_i(0) = \text{logsig}(n_i(0)) = \dfrac{1}{1+e^{-n_i(0)}}$; $\Rightarrow (1-a_i)a_i = \dfrac{e^{-p_i}}{1+e^{-p_i}}$

$$n(0) = W(0)\,p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \Rightarrow a(0) = \begin{bmatrix} \frac{1}{1+e^{-p_1}} \\ \frac{1}{1+e^{-p_2}} \end{bmatrix}$$

$$\therefore \quad (t-a)\Big|_{k=0} = \begin{bmatrix} t_1 - \frac{1}{1+e^{-p_1}} \\ t_2 - \frac{1}{1+e^{-p_2}} \end{bmatrix}$$

$$\Rightarrow \quad s(0) = -2\begin{bmatrix} \frac{e^{-p_1}}{1+e^{-p_1}} & 0 \\ 0 & \frac{e^{-p_2}}{1+e^{-p_2}} \end{bmatrix}\begin{bmatrix} t_1 - \frac{1}{1+e^{-p_1}} \\ t_2 - \frac{1}{1+e^{-p_2}} \end{bmatrix}$$

as $W(0) = I$

$$W(1) = I_2 + 2\alpha\begin{bmatrix} \frac{e^{-p_1}}{1+e^{-p_1}} & 0 \\ 0 & \frac{e^{-p_2}}{1+e^{-p_2}} \end{bmatrix}\begin{bmatrix} t_1 - \frac{1}{1+e^{-p_1}} \\ t_2 - \frac{1}{1+e^{-p_2}} \end{bmatrix}\begin{bmatrix} p_1 & p_2 \end{bmatrix}$$