EE434

$$p \xrightarrow{W} n \to f(\cdot) \to a$$

$p_i,\ a_i \in \{-1, 1\}$; $1 \le i \le S$

component

autoassociative

$W = PP^T$; $Wp = n$; $\underline{a}_i = \underline{p}_i$; $P = [\underline{p}_1, \dots, \underline{p}_Q]$

vector

$\underline{p}_i^T \underline{p}_j = 0$ if $i \neq j$

= size of p = dimension

$$\underline{p}_i = \begin{bmatrix} p_{i1} \\ p_{i2} \\ \vdots \\ p_{iS} \end{bmatrix}$$

Example $S = 12$

choose $\underline{p}_1 = [1\ 1\ 1\ 1\ 1\ 1 \mid 1\ 1\ 1\ 1\ 1\ 1]^T$

$\underline{p}_2 = [1\ 1\ 1\ 1\ 1\ 1\ {-1}\ {-1}\ {-1}\ {-1}\ {-1}\ {-1}]^T$

find an $\underline{x}$ such, $\underline{p}_1^T \underline{x} = 0$, $\underline{p}_2^T \underline{x} = 0$

$$\begin{bmatrix} \underset{\sim}{\varphi}_1^T \\ \underset{\sim}{\varphi}_2^T \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1&1&1&1 & 1&1&1&1 & 1&1&1&1 \\ 1&1&1&1 & 1&1&-1&-1 & -1&-1&-1&-1 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

subtract 1st row from second ⇒

$$\begin{bmatrix} 1&1&1&1 & 1&1&1&1&1&1&1&1 \\ 0&0&0&0 & 0&0&-2&-2&-2&-2&-2&-2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \xrightarrow{\text{divide 2nd by } -2} \left[\begin{array}{cccccc|cccccc} 1&1&1&1&1&1 & 1&1&1&1&1&1 \\ 0&0&0&0&0&0 & 1&1&1&1&1&1 \end{array}\right] x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

note that will work $x = \left[\begin{array}{cccccc|cccccc} 1&1&1&-1&-1&-1 & 1&-1&1&-1&1&-1 \end{array}\right]^T = \underset{\sim}{\varphi}_3$

Now form $W = \begin{bmatrix} \underset{\sim}{\varphi}_1 & \underset{\sim}{\varphi}_2 & \underset{\sim}{\varphi}_3 \end{bmatrix} \begin{bmatrix} \underset{\sim}{\varphi}_1 & \underset{\sim}{\varphi}_2 & \underset{\sim}{\varphi}_3 \end{bmatrix}^T = \begin{bmatrix} \underset{\sim}{\varphi}_1 & \underset{\sim}{\varphi}_2 & \underset{\sim}{\varphi}_3 \end{bmatrix} \begin{bmatrix} \underset{\sim}{\varphi}_1^T \\ \underset{\sim}{\varphi}_2^T \\ \underset{\sim}{\varphi}_3^T \end{bmatrix}$

$$W = \begin{bmatrix} 1&1&1 \\ 1&1&1 \\ 1&1&1 \\ 1&1&1 \\ 1&1&-1 \\ 1&1&-1 \\ 1&1&-1 \\ 1&-1&1 \\ 1&-1&-1 \\ 1&-1&1 \\ \vdots&\vdots&\vdots \end{bmatrix} \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&-1&-1&-1&-1&-1&-1 \\ 1&1&1&-1&-1&-1&1&-1&1&-1&1&-1 \end{bmatrix} = \begin{bmatrix} 3&3&3&1&1&1+1&1+1&-1&1&-1&1&-1 \\ 3&3&&&&&&&&&& \\ 3&&3&&&&&&&&& \\ 1&&&\ddots&&&&&&&& \\ 1&&&&&&&&&&& \end{bmatrix}$$

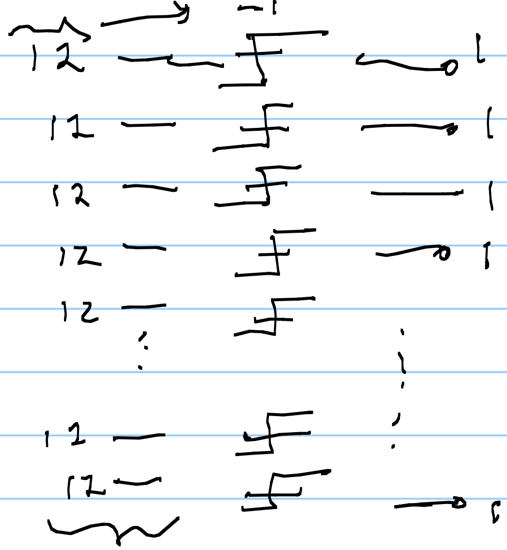
$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$W = W^T = P P^T$$

$$(PP^T)^T = (P^T)^T P^T = PP^T \qquad \underset{\sim}{p}_1^T \underset{\sim}{p}_1 = \sum_{i=1}^{12} 1 \cdot 1 = 12$$

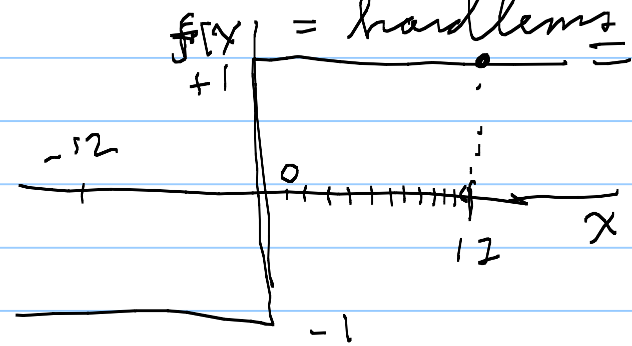
$$W \underset{\sim}{p}_1 = \begin{bmatrix} \underset{\sim}{p}_1 & \underset{\sim}{p}_2 & \underset{\sim}{p}_3 \end{bmatrix} \begin{bmatrix} \underset{\sim}{p}_1^T \\ \underset{\sim}{p}_2^T \\ \underset{\sim}{p}_3^T \end{bmatrix} \underset{\sim}{p}_1 = \begin{bmatrix} \underset{\sim}{p}_1 & \underset{\sim}{p}_2 & \underset{\sim}{p}_3 \end{bmatrix} \begin{bmatrix} \underset{\sim}{p}_1^T \underset{\sim}{p}_1 \\ \underset{\sim}{p}_2^T \underset{\sim}{p}_1 \\ \underset{\sim}{p}_3^T \underset{\sim}{p}_1 \end{bmatrix} = \begin{bmatrix} \underset{\sim}{p}_1 & \underset{\sim}{p}_2 & \underset{\sim}{p}_3 \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} = 12 \underset{\sim}{p}_1$$

$\therefore$ $n = 12 \underset{\sim}{p}_1$ when $\underset{\sim}{p}_1$ = input



$$\underset{\sim}{p}_2^T \underset{\sim}{p}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 6 + (-6) = 0$$

$f(x)$ = hardlims



$\underset{\sim}{x}$ $\quad f(\underline{x})$ $\quad \underline{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ \\ \\ \end{bmatrix}$

know p. 11-25