EE 434

Note Title 3/11/2004

$$C\frac{dn}{dt} = Wa + I_b - Gn \quad ; \quad a = f(n) = \begin{bmatrix} f_1(n_1) \\ \vdots \\ f_S(n_S) \end{bmatrix}$$

$$\frac{da}{dt} = \frac{df(n)}{dn}\cdot\frac{dn}{dt} \quad ; \quad \frac{df(n)}{dn} = \begin{bmatrix} df_1/dn_1 & 0 & \cdots & 0 \\ 0 & df_2/dn_2 & \cdots & 0 \\ \vdots & & & \\ 0 & & & df_S/dn_S \end{bmatrix}$$

assume $\left[\frac{df(n)}{dn}\right]^{-1}$ exists ; true if have monotonic increasing $f(\cdot)$'s.

Look $f(x) = \frac{1}{1+e^{-x}}$; $\frac{df}{dx} = \frac{-1}{(1+e^{-x})^2}\times(-e^{-x}) = \frac{e^{-x}}{(1+e^{-x})^2}$

Let $f(x) = y = \frac{1}{1+e^{-x}} \Rightarrow e^{-x}y + y = 1 \Rightarrow e^{-x} = \frac{1-y}{y} = \frac{1}{y} - 1$

$$\frac{df}{dx} = \frac{dy}{dx} = \frac{\left(\frac{1}{y}-1\right)}{\left(\cancel{1} + \frac{1}{y} - \cancel{1}\right)^2} = y^2\left(\frac{1-y}{y}\right) = y\cdot(1-y) \Rightarrow \frac{d\,f(x)}{dx} = f(x)(1-f(x))$$

$$C\left[\frac{df}{dx}\right]^{-1}\cdot\frac{da}{dt} = Wa + I_b - G\cdot f^{-1}(a)$$ Hopfield diff eq.

given $a(0)$ desire $a(t) \to$ desired memory a's.

want these as equilibria (where $da/dt = 0$)

desire a's to satisfy

$$0 = Wa + I_b - G f^{-1}(a)$$

∴ choose a's as desire & find W, I_b, G so satisfy ⊛

choose $a_0, a_1, a_2 \ldots a_M \Leftarrow$

$$0 - 0 = W(a_1 - a_0) + I_b - I_b - G f^{-1}(a_1) + G f^{-1}(a_0)$$

$$\Rightarrow \quad 0 = W(a_1 - a_0) - G[f^{-1}(a_1) - f^{-1}(a_0)]$$

$$0 = W(a_2 - a_0) - G[f^{-1}(a_2) - f^{-1}(a_0)]$$

do for as many eqs as rows & columns in $W_{ij} = S \Rightarrow M = S$

allows to get $WA - GB$, $A = \left[a_1 - a_0 \;\vdots\; a_2 - a_0 \;\vdots\; \ldots\; a_S - a_0\right]$

$WA = GB$

$$\Rightarrow \boxed{W = GBA^{-1}}$$

$B = \left[f^{-1}(a_1) - f^{-1}(a_0) \;\vdots\; \ldots \;\vdots\; f^{-1}(a_S) - f^{-1}(a_0)\right]$

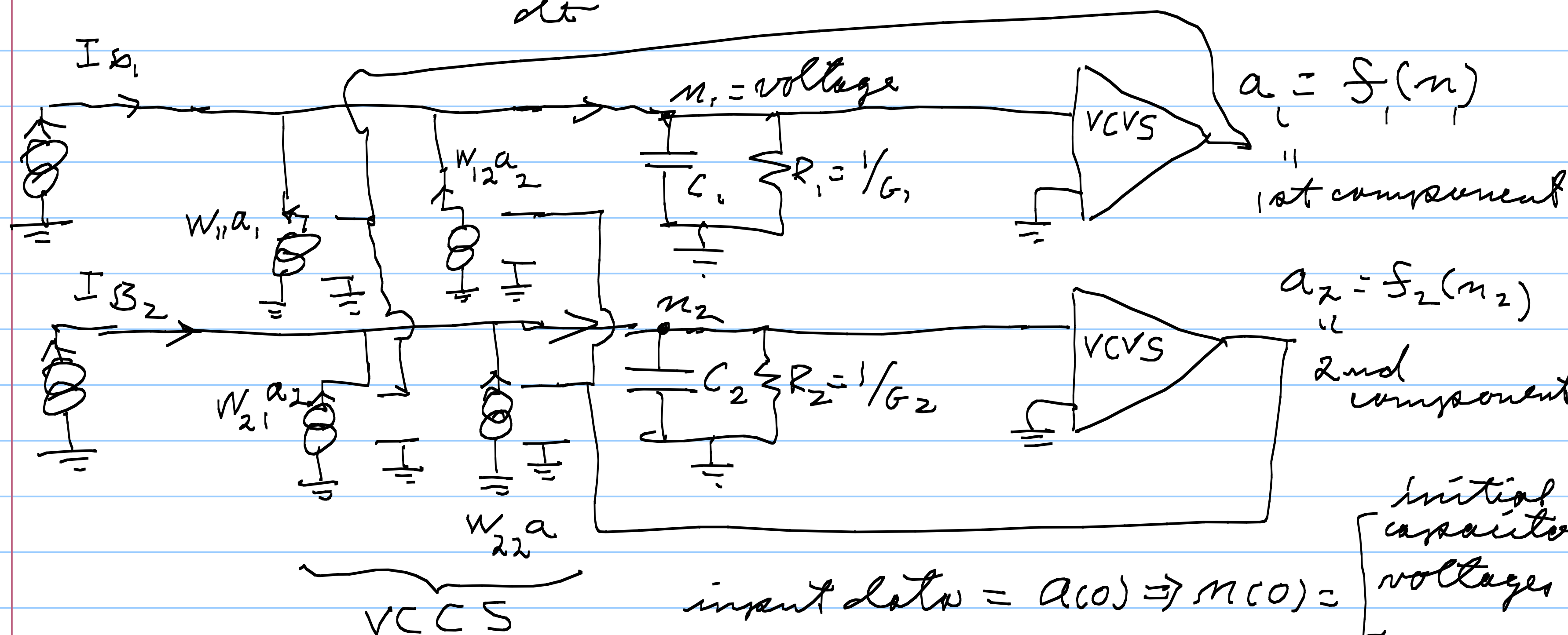
needs A nonsingular

i.e. $a_i - a_0$ are linearly independent then $I_b = -Wa_0 + G f^{-1}(a_0)$ ∴ know W & I_b

we needed $W = W^T$ for using the Lyapunov function

$$V(a) = -\frac{1}{2} a^T W a - I_b^T a + \sum_{i=1}^{S} G_{ii} \int_0^{a_i} f^{-1}(x_i)\,dx_i$$

$$V(a) \geq 0, \quad \frac{dV(a)}{dt} \leq 0$$



input data = $a(0) \Rightarrow x(0) =$ [initial capacitor voltages] we are guaranteed when we run this system to go to one of $a_0, a_1, \ldots, a_S$ after a "long enough" if $W = W^T$ by leaving a_0 free & solving $W = W^T$ can guarantee $W = W^T$ (uses a nonlinear equation in a_0)