EE 434

Note Title 3/9/2004

Chapter 18 Dynamic Hopfield NN

$$\underbrace{C\frac{dn}{dt}}_{\text{capacitors}} = \underbrace{Ta}_{\substack{T \\ \text{feedback}}} - \underbrace{Gn}_{\text{resistors}} + \underbrace{I_b}_{\text{bias}} \quad ; \quad a = f(n)$$

need $n(0)$

$f(\cdot)$ is an S-vector of activation functions

n, a, I_b are S-vectors; G, C $S \times S$ diagonal, > 0 on diagonal

T is $S \times S$

desire $T = T^T$, real

if a = voltage, T would be a VCCS

T = like weights

VCCS = voltage controlled current source

n is like a voltage

$f_i(\cdot)$ are monotonic increasing & bounded

equilibrium points are "signals" want to recognize

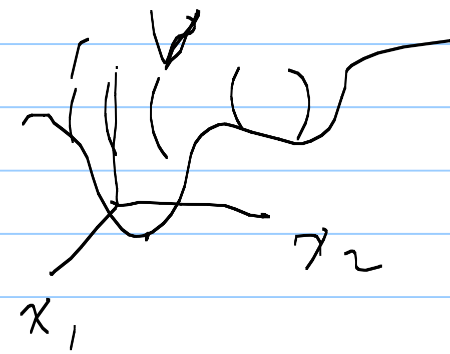
$\frac{dx}{dt} = 0 \Rightarrow$ equilibrium = rest point

$$= 0 = Ta - Gx + I_b = Ta - Gf^{-1}(a) + I_b = 0$$

look at the class called "gradient"

$$\frac{dx}{dt} = -\nabla_x V(x) \quad ; \quad V(x) \geq 0$$

scalar energy function



$$\frac{dV(x)}{dt} = \nabla_x V(x)^T \cdot \frac{dx}{dt}$$

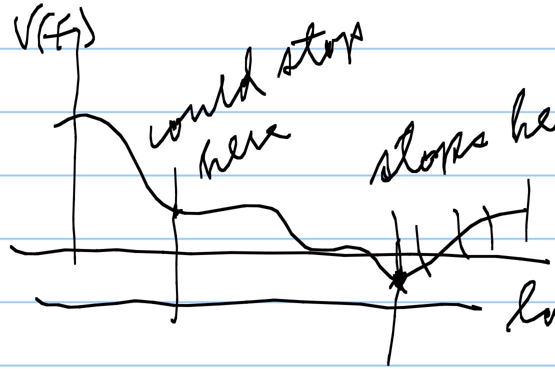
$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_S \end{bmatrix}$$

$$\nabla_x V(x) = \begin{bmatrix} \partial V / \partial x_1 \\ \partial V / \partial x_2 \\ \vdots \end{bmatrix}$$

$\left[\partial V / \partial x_S\right]$

If $\frac{dx}{dt} = -\nabla_x V(x)$ then $\frac{dV(x)}{dt} = -\nabla_x V(x)^T \cdot \nabla_x V(x) = -$(sum of squares) ≤ 0

this means $V(x)$ will always decrease in time; if bounded below then has to come to a minimum.



p. 18-5 eq. (18.8)

$$V(a) = -\frac{a^T T a}{2} - I_b^T a + \sum_{i=1}^{S} G_{ii} \int_0^{a_i} f_i^{-1}(q)\,dq$$

$$\nabla_a V(a) = -\frac{T^T a}{2} - \frac{T a}{2} - I_b + G f^{-1}(a) = -\left(\frac{T^T + T}{2}\right) a - I_b + Gn$$

desire $= -(Ta + b - Gm)$ but $\frac{T^T + T}{2} \neq T$ in general

but it is if $T = T^T$

so in a design want $T = T^T$ in book is ok @ (18.68) p. 18-18 where

$$T = \sum p_i p_i^T$$

They use $I_b = 0_S$, $G = 0_{S\times S}$

$$V(a)_{\text{1st term}} = -a^T T a = -\left[\sum_{i=1}^{S} a_i \left(\sum_{j=1}^{S} T_{ij} a_j\right)\right]$$

$$\frac{\partial V(a)}{\partial a_k} = -\sum_{i=1}^{S} \delta_{ik}\left(\sum_{j=1}^{S} T_{ij} a_j\right) - \sum_{i=1}^{S} a_i \left(\sum_{j=1}^{S} T_{ij}\delta_{jk}\right)$$

$$= -\sum_{j=1}^{S} T_{kj} a_j - \sum_{i=1}^{S} a_i T_{ik} = -\left\{\sum_{j=1}^{S} T_{kj} a_j + \sum_{i=1}^{S} T_{ik} a_i\right\}$$

$$= -\left(\sum_{i=1}^{S} T_{ki}\, a_i + T_{ik}\, a_i\right)$$

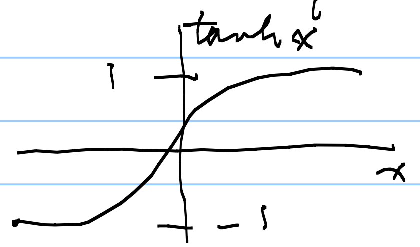
$$\frac{\partial V}{\partial a} = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix} = -(T + T^T)\, a$$

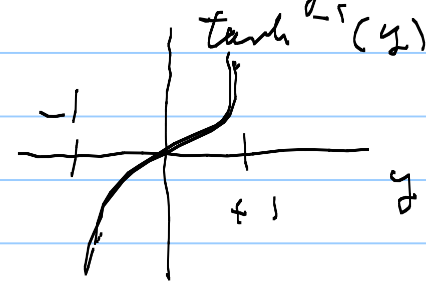
desire solutions of $C\frac{dn}{dt} = C\frac{d f^{-1}(a)}{dt} = Ta + I_b - G f^{-1}(a)$

create so equilibria are data points

i.e. desire a such that $\underline{0} = Ta + I_b - G f^{-1}(a)$

choose, $f_i(\cdot)$ = activation functions; for example $\tanh(n_i) = f_i(n_i)$





both tanh & $\tanh^{-1}$ are differentiable & monotone increasing.

so choose $S+1$ vectors with entries in the open set $(-1, 1)$