ENEE 434 Spring 2003

Homework #3 Solutions

E7.3

$$P_1 = [\,1 \quad -1 \quad -1 \quad 1 \quad 1 \quad -1\,]^T \qquad t_1 = 1$$
$$P_2 = [\,1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1\,]^T \qquad t_2 = -1$$

$$W = T P^T = [1 \quad -1] \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix}$$

$$= [\,0 \quad -2 \quad 0 \quad 0 \quad 2 \quad -2\,]$$

For the two patterns given:

$$a_1 = \text{hardlims}(W P_1) = \text{hardlims}(2+2+2) = 1$$
$$a_2 = \text{hardlims}(W P_2) = \text{hardlims}(-2-2-2) = -1$$

E8.2 i. $F(x)\big|_{x=\begin{bmatrix}0\\0\end{bmatrix}} = e^{10}$

$$\frac{\partial F(x)}{\partial x_1}\bigg|_{x=\begin{bmatrix}0\\0\end{bmatrix}} = e^{(2x_1^2+2x_2^2+x_1-5x_2+10)}(4x_1+1) = e^{10}$$

$$\frac{\partial F(x)}{\partial x_2}\bigg|_{x=\begin{bmatrix}0\\0\end{bmatrix}} = e^{(2x_1^2+2x_2^2+x_1-5x_2+10)}(4x_2-5) = -5e^{10}$$

$$\nabla F(x)\big|_{x=\begin{bmatrix}0\\0\end{bmatrix}} = \begin{bmatrix} e^{10} \\ -5e^{10} \end{bmatrix}$$

$$\nabla^2 F(x)\big|_{x=\begin{bmatrix}0\\0\end{bmatrix}} = \begin{bmatrix} 5e^{10} & -5e^{10} \\ -5e^{10} & 29e^{10} \end{bmatrix}$$

$$F_a(x) = e^{10}\left(1 + [1 \quad -5]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2}[x_1 \quad x_2]\begin{bmatrix} 5 & -5 \\ -5 & 29 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$$

ii. $x_a^* = -A^{-1}d$

$$= -\left(e^{10}\begin{bmatrix} 5 & -5 \\ -5 & 29 \end{bmatrix}\right)^{-1} e^{10}\begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{30} \\ \frac{1}{6} \end{bmatrix}$$

iii. $\nabla F(x)\Big|_{x=x^*} = 0$

$$\begin{bmatrix} 4x_1^* + 1 \\ 4x_2^* - 5 \end{bmatrix} = 0 \qquad x^* = \begin{bmatrix} -\frac{1}{4} \\ \frac{5}{4} \end{bmatrix}$$

iv. $F(x)$ and $F_a(x)$ are actually different functions, therefore, the difference between the two stationary points is reasonable.

E8.4 $\quad F(x) = x^4 - \frac{1}{2}x^2 + 1$

i.

$$\frac{dF(x)}{dx} = 4x^3 - x = x(4x^2 - 1) = 0$$

$x_1 = 0, \quad x_2 = -\frac{1}{2}, \quad x_3 = \frac{1}{2}$

ii. $$\frac{d^2F(x)}{dx^2} = 12x^2 - 1$$

| | | |
|---|---|---|
| For $x_1 = 0$ | $12(0) - 1 = -1$ | maximum point |
| $x_2 = -\frac{1}{2}$ | $12(\frac{1}{4}) - 1 = 2$ | minimum point |
| $x_3 = \frac{1}{2}$ | $12(\frac{1}{4}) - 1 = 2$ | minimum point |