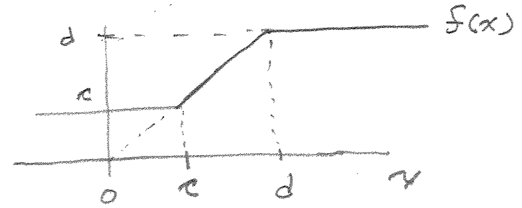
EE 434 final
solution 05/19/03

#3.

a)



b)

$$f(x) = \alpha + \beta \,\text{satlin}(\gamma x + \delta) = \begin{cases} \alpha & \gamma x + \delta < 0 \equiv x < -\delta/\gamma \\ \alpha + \beta & \gamma x + \delta > 1 \equiv x > (1-\delta)/\gamma \\ \text{linear in between} & \end{cases}$$

$$\doteq \begin{cases} c & \text{for } x < c \\ d & \text{for } x > d \\ \text{linear in between } \left(\text{slope} = 1 = \frac{d-c}{d-c}\right) & \end{cases}$$

identifying values and ranges

$\alpha = c, \quad c = -\delta/\gamma$

$\alpha + \beta = d \Rightarrow \beta = d - c \quad$ for $d = (1-\delta)/\gamma$

$\Rightarrow d = \frac{1}{\gamma} + c \Rightarrow \gamma = \frac{1}{d-c}$

$\Rightarrow \delta = -\gamma c = -\frac{c}{d-c}$

$\therefore$

$$\underline{\underline{f(x) = c + (d-c)\,\text{satlin}\left(\frac{x-c}{d-c}\right)}}$$

c_1) $\frac{dx}{dt} = -x - Wy, \quad y = F(x)$ with $F_i(x) = f(x_i)$

c_2) The normal choice of energy function would give the Lyapunov function if $W = W^T$

$$\mathcal{E}(y) = \frac{1}{2} y^T W y + \sum_{i=1}^{n} \left[1 \cdot \int_c^{y_i} F_i^{-1}(x_i)\, dx_i \cdot 1(c < y_i < d) + c y_i 1(c - y_i) + d\, 1(y_i - d) \right.$$

$$d\mathcal{E}(y)/dt = \dot{y}^T (Wy + x) = -(x + Wy)^T \left[dF_i/dx_j \right] (x + Wy) \le 0$$

or have stability; if $W \ne W^T$ also true by direct calculation

c_3) Equilibrium of $dx/dt = \underline{0} = -x - Wy$

$x_{eq} = -WF(x_{eq})$

If $c < x_i < d$ for all i, $x_{eq} = -W x_{eq}$

$\Rightarrow x_{eq}$ = eigenvector for unity eigenvalue of $-W$

otherwise x_i decreases to c or increases to d

(as stable with $d\mathcal{E}(y) \to 0$)