file: c:\math\mcad80\rwn_mcad\backprop_ex1.mcd   RWN   02/19/02
Example of backpropagation for 2-layer network

Assume a two layer single input single output network with two tansig neurons in the first layer and one purelin neuron in the output layer to approximate 3cos(p); train on p=2.5=a0.

$a0 := 2.5$

Choose initial weights and biases;

For first layer

$W1 := \begin{bmatrix} 0.2 \\ -0.5 \end{bmatrix} \qquad b1 := \begin{bmatrix} -0.3 \\ 0.8 \end{bmatrix}$

For second layer

$W2 := (-2 \quad 0.5) \qquad b2 := -0.7$

The training ouput is 3cos(2.5)

$t := 3 \cdot \cos(2.5) \qquad t = -2.403$

The network functions are

$n1 := W1 \cdot a0 + b1 \qquad n1 = \begin{bmatrix} 0.2 \\ -0.45 \end{bmatrix} \qquad n1_1 = 0.2 \quad n1_2 = -0.45$

$a1 := \tanh(n1) \qquad a1 = \begin{bmatrix} 0.197 \\ -0.422 \end{bmatrix} \qquad a1_1 = 0.197 \quad a1_2 = -0.422$

$n2 := W2 \cdot a1 + b2 \qquad n2 = (-1.306)$

$a2 := n2$ We wish this to become t, that is, n2==>t is desired by training

Output difference, e, and error E $e := t - a2 \qquad e = (-1.098)$

$E := e^T \cdot e \qquad E = (1.205)$

The function derivatives are found from y=tanh(x)=(2/(1+e^(-2x))-1) as dy/dx=(1-y)(1+y), while for the second layer it is the identity. Thus

$da1(y) := (1 - y)\,(1 + y) \qquad da1\left(a1_1\right) = 0.961$

$da1\left(a1_2\right) = 0.822$

$df1 := \begin{bmatrix} da1\left(a1_1\right) & 0 \\ 0 & da1\left(a1_2\right) \end{bmatrix} \qquad df1 = \begin{bmatrix} 0.961 & 0 \\ 0 & 0.822 \end{bmatrix}$

$df2 := 1$

Start the backpropagation

$s2 := -1 \cdot df2 \cdot (t - a2)$ $\quad$ $s2 = (1.098)$

$s1 := df1 \cdot W2^T \cdot s2$ $\quad$ $s1 = \begin{bmatrix} -2.11 \\ 0.451 \end{bmatrix}$

Weight update; using a learning rate α=0.3 overshoots so choose smaller

$\alpha := 0.1$

$W2new := W2 - \alpha \cdot s2 \cdot a1^T$ $\quad$ $W2new = (-2.022 \quad 0.546)$

$b2new := b2 - \alpha \cdot s2$ $\quad$ $b2new = (-0.81)$

$W1new := W1 - \alpha \cdot s1 \cdot a0$

Really want a transpose on a0 but MathCad won't accept on his scalar

$b1new := b1 - \alpha \cdot s1$

$W1new = \begin{bmatrix} 0.727 \\ -0.613 \end{bmatrix}$

$b1new = \begin{bmatrix} -0.089 \\ 0.755 \end{bmatrix}$

$n1new := W1new \cdot a0 + b1$

$a1new := \tanh(n1new)$ $\quad$ $a1new = \begin{bmatrix} 0.908 \\ -0.624 \end{bmatrix}$

$n2new := W2new \cdot a1new + b2new$

$a2new := n2new$ $\quad$ $enew := t - a2new$ $\quad$ $Enew := enew^T \cdot enew$

$n1new = \begin{bmatrix} 1.519 \\ -0.732 \end{bmatrix}$ $\quad$ $a1new = \begin{bmatrix} 0.908 \\ -0.624 \end{bmatrix}$

$n2new = (-2.987)$ $\quad$ $a2new = (-2.987)$

$enew = (0.584)$ $\quad$ compare to previous $\quad$ $e = (-1.098)$

$Enew = (0.341)$ $\quad$ $E = (1.205)$