ENEE434 March 19, 2002 WS/TH
Solution to midterm exam

Problem 1

$a_1(t) = iw_{1,1}\, p(t) + iw_{1,2}\, p(t-1) = 2\, p(t) + p(t-1)$
$a_2(t) = lw_{2,1}\, a_1(t) + lw_{2,2}\, a_1(t-1) = a_1(t) + 2\, a_1(t-1)$

$t=0 : a_1(0)\ ,\ a_2(t) = 0$
$t=1 : a_1(1) = 2\, p(1) + p(0) = 2*2 = 4$
$\quad a_2(1) = a_1(1) + 2\, a_1(0) = 4$
$t=2 : a_1(2) = 2\, p(2) + p(1) = 2*1 + 2 = 4$
$\quad a_2(2) = a_1(2) + 2\, a_1(1) = 4 + 2*4 = 12$
$t=3 : a_1(3) = 2\, p(3) + p(2) = 2*4 + 1 = 9$
$\quad a_2(3) = a_1(3) + 2\, a_1(2) = 9 + 2*4 = 17$
$t=4 : a_1(4) = 2\, p(4) + p(3) = 2*3 + 4 = 10$
$\quad a_2(4) = a_1(4) + 2\, a_1(3) = 10 + 2*9 = 28$
$t=5 : a_1(5) = 2\, p(5) + p(4) = 2*2 + 3 = 7$
$\quad a_2(5) = a_1(5) + 2\, a_1(4) = 7 + 2*10 = 27$

The outputs will cyclicly repeat after t = 6.
The results can be checked by MatLab as follow:

```
>> net1 = newlin([-5 5], 1, [0 1]);
>> net2 = newlin([-5 5], 1, [0 1]);
>> net1.IW{1,1} = [2 1];
>> net2.IW{1,1} = [1 2];
>> p = {2 1 4 3 2};
>> a1 = sim(net1, p)

a1 =

   [4]    [4]    [9]    [10]    [7]

>> a2 = sim(net2, a1)

a2 =

   [4]    [12]    [17]    [28]    [27]

>> p = {2 1 4 3 2 1 4 3 2 1 4 3};
>> a1 = sim(net1, p)

a1 =

  Columns 1 through 8

   [4]    [4]    [9]    [10]    [7]    [4]    [9]    [10]

  Columns 9 through 12
```

```
   [7]    [4]    [9]    [10]

>> a2 = sim(net2, a1)

a2 =

  Columns 1 through 7

    [4]    [12]    [17]    [28]    [27]    [18]    [17]

  Columns 8 through 12

    [28]    [27]    [18]    [17]    [28]
```

Problem 2

$\mathbf{p} = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 2 & 1 \end{bmatrix}$, $\mathbf{t} = [0\ 1\ 1]$, $\mathbf{W}(0) = [0\ 0]$, $\mathbf{b}(0) = [0\ 0\ 0]$

First epoch:

$\mathbf{a} = hardlim\ (\mathbf{W}(0)\ \mathbf{p} + \mathbf{b}(0))$

$= hardlim\ ([0\ 0\ 0]) = hardlim\ ([0\ 0\ 0]) = [1\ 1\ 1]$

$\mathbf{e} = \mathbf{t} - \mathbf{a} = [0\ 1\ 1] - [1\ 1\ 1] = [-1\ 0\ 0]$

$\mathbf{W}^{new} = \mathbf{W}^{old} + \mathbf{e}\mathbf{p}^T = [0\ 0] + [-1\ 2] = [-1\ 2] = \mathbf{W}(1)$

$\mathbf{b}^{new} = \mathbf{b}^{old} + \mathbf{e} = [-1\ 0\ 0] = \mathbf{b}(1)$

Second epoch:

$\mathbf{a} = hardlim\ (\mathbf{W}(1)\ \mathbf{p} + \mathbf{b}(1))$

$= hardlim\ ([-1\ \ 2] \begin{bmatrix} 1 & -3 & 2 \\ -2 & 2 & 1 \end{bmatrix} + [-1\ 0\ 0]) = hardlim\ ([-5\ 7\ 0]) = [0\ 1\ 1]$

$\mathbf{e} = \mathbf{t} - \mathbf{a} = [0\ 1\ 1] - [0\ 1\ 1] = [0\ 0\ 0]$

$\mathbf{W}^{new} = \mathbf{W}^{old} + \mathbf{e}\mathbf{p}^T = [0\ 0] + [-1\ 2] = [-1\ 2] = \mathbf{W}(2)$

$\mathbf{b}^{new} = \mathbf{b}^{old} + \mathbf{e} = [-1\ 0\ 0] = \mathbf{b}(2)$

The performance goal is met, and it takes two epochs to reach the goal.
The MatLab results and the error rate plot is shown below.

```
>> net = newp([-5 5; -5 5], 1);
>> net.trainParam.epochs = 3;
>> p = [[1;-2] [-3;2] [-2;1]];
>> t  = [0 1 1];
>> net = train(net, p, t)
TRAINC, Epoch 0/3
TRAINC, Epoch 2/3
```

TRAINC, Performance goal met.