Solution to Problem 2 - Homework 4 - ENEE 302

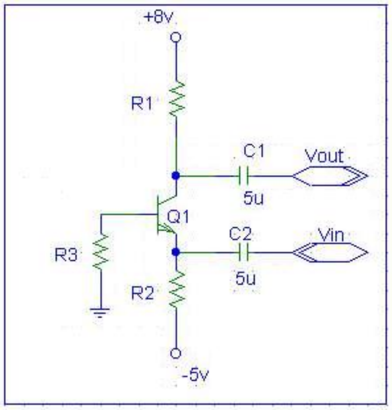
AD Revised 05/14/02 & 05/19/02

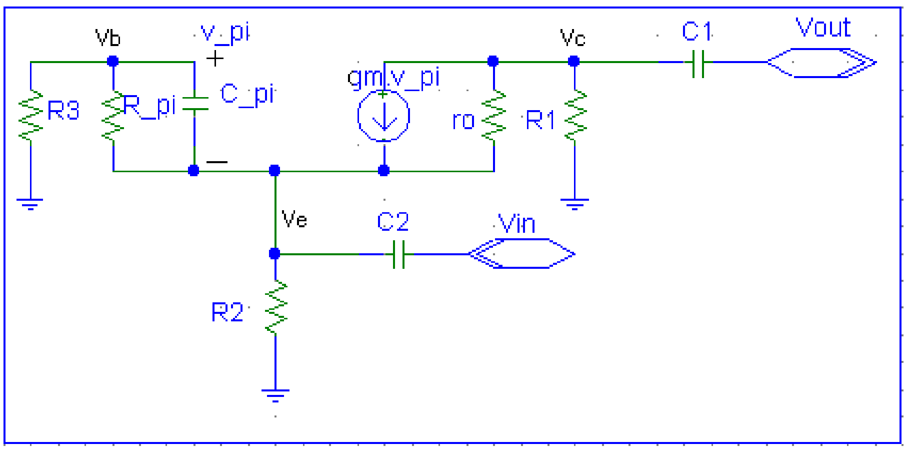
2. For the following common-base circuit:

a) Determine the gain Av(s) = Vout / Vin for all frequencies assuming $C_\mu = 0$.

b) Determine Zi and Zo, input and output impedances for all frequencies as functions of s.

c) Plot Av(s) using Pspice and the BN2x2 npn transistor for R1=3.6K, R2=3.9K, R3 = 390K, C1= 5uF, C2=5uF.





Above, is the small signal model of the circuit.

Here, $r_\pi = \frac{\beta \times Ic}{V_T}, r_o = \frac{V_A}{I_C}, g_m = \frac{I_C}{V_T}$ and C_π depends on the transistor parameters.

where I_C, the DC bias current, is calculated from the basic KVL in the circuit around the base-emitter-V_{EE} loop:

$$0 = R_3 \times \frac{I_C}{\beta} + 0.7 + \frac{I_C}{\alpha} \times R_2 - 5.0$$

The impedance of capacitor C will be considered $\frac{1}{C \times s}$ in writing KVL and KCL equations since the behavior of the circuit in all frequencies is wanted.

* Below, are the basic KVL and KCL equations required to calculate Zi :

KVL @ c: $\frac{v_c}{R_1}+\frac{v_c-v_e}{r_o}+g_m(v_b-v_e)=0$; C_1 is connected to ∞ load!

KVL @ b: $\frac{v_b-v_e}{r_\pi \parallel c_\pi}+\frac{v_b}{R_3}=0$

KVL @ e: $(v_e-v_{in})\times c_2 s+\frac{v_e-v_b}{r_\pi \parallel c_\pi}+\frac{v_e}{R_2}+\frac{v_e-v_c}{r_o}-g_m(v_b-v_e)=0$

Assuming $r_o \geq R_1$ and $i_e \approx \beta \times i_b$, the followings can be obtained after simplifying the circuit equations:

The input impedance is $\frac{v_{in}}{I_{c2}}=\frac{v_{in}}{(v_{in}-v_e)\times c_2 s}$, which after simplification will be the following: Zi = $\frac{1}{C_2\times s}+(R_2 \parallel [(\frac{r_\pi}{\beta}\parallel c_\pi)+\frac{R_3}{\beta}])$;

Note that this impedance is the series of $\frac{1}{C_2\times s}$ with the impedance that is seen looking from C_2 in the circuit, which is.

$R_2 \parallel [(\frac{r_\pi}{\beta}\parallel c_\pi)+\frac{R_3}{\beta}]$.

* Below, are the basic KVL and KCL equations required to calculate Zo :

KVL @ c: $\frac{v_c-v_o}{R_1}+\frac{v_c-v_e}{r_o}+g_m(v_b-v_e)=0$

KVL @ b: $\frac{v_b-v_e}{r_\pi \parallel c_\pi}+\frac{v_b}{R_3}=0$

KVL @ e: $v_e\times c_2 s+\frac{v_e-v_b}{r_\pi \parallel c_\pi}+\frac{v_e}{R_2}+\frac{v_e-v_c}{r_o}-g_m(v_b-v_e)=0$; Note Vin = 0.

Assuming $r_o \geq R_1$ and $i_e \approx \beta \times i_b$, the followings can be obtained after simplifying the circuit equations:

The output impedance is $\frac{v_{out}}{I_{c1}}=\frac{v_{out}}{(v_{out}-v_c)\times c_1 s}$, which after simplification will be the following: Zo = $\frac{1}{C_1\times s}+R_1$

Note the output impedance is the series of $\frac{1}{C_1\times s}$ with the impedance that is seen looking from C_1 in the circuit which is $R1$ with a good approximation. ($r_o \geq R_1$)

* Below, are the basic KVL and KCL equations required to calculate Av :

KVL @ c: $\frac{v_c}{R_1} + \frac{v_c - v_e}{r_o} + g_m(v_b - v_e) + (v_o - v_c) \times C_1 s = 0$

KVL @ b: $\frac{v_b - v_e}{r_\pi \parallel c_\pi} + \frac{v_b}{R_3} = 0$

KVL @ e: $(v_e - v_{in}) \times C_2 s + \frac{v_e - v_b}{r_\pi \parallel c_\pi} + \frac{v_e}{R_2} + \frac{v_e - v_c}{r_o} - g_m(v_b - v_e) = 0$

The following equations can be rearranged to solve for base, collector and emitter voltages:

$$\begin{cases} g_m v_b + (\frac{1}{R_1} + \frac{1}{r_o} - C_1 s)v_c + (\frac{-1}{r_o} - g_m)v_e = C_1 s \times v_o \\ (\frac{1}{r_\pi \parallel c_\pi} + \frac{1}{R_3})v_b + (\frac{-1}{r_\pi \parallel c_\pi})v_e = 0 \\ (\frac{-1}{c_\pi \parallel r_\pi} - g_m)v_b + (\frac{-1}{r_o})v_c + (C_2 s + \frac{1}{r_\pi \parallel c_\pi} + \frac{1}{R_2} + \frac{1}{r_o} + g_m)v_e = C_2 s \times v_{in} \end{cases}$$

Finally, below is the expression for gain:

$$A_v(s) = \frac{-R_2}{R_1} \times \frac{R_1 + r_0 + r_o R_1 C_1 s}{r_o R_1 C_1 s} \times \frac{C_2 s(R_3(r_\pi + c_\pi) + r_\pi c_\pi R_2)}{\beta R_2(r_\pi + c_\pi) + R_3(r_\pi + c_\pi) + r_\pi c_\pi + C_2 s(R_3(r_\pi + c_\pi) + r_\pi c_\pi R_2)}$$

Note for high frequencies, the second and third term get close to one and the gain is close to –R1/R2.

Note that initially assuming C1 and C2 short (very high frequencies), then the gain will be like a normal common-base amplifier:

$A_v \cong -\frac{R_1}{R_2}$, but for low frequencies C1 and C2 behave like open circuit and the gain will be zero.

The cut-off frequency where the gain will rise from 0 is calculated as below:

$$w_1 = \frac{1}{2\pi \times C_1 \times R_1}$$

$$w_2 = \frac{1}{2\pi \times C_2 \times (R_2 \parallel (r_e + \frac{R_3}{\beta}))}$$

w_1 is the frequency associated with C_1 and is calculated by multiplying these effective R.C seen from C_1 and so is w_2 calculated for C_2.

So, the overall gain expression will be the following:

$$A_v(jw) = \begin{cases} 0 & w < \min(w_1, w_2) \\ -\dfrac{R_1}{R_2} & w > \min(w_1, w_2) \end{cases}$$