Solutions 610 final Fall 2020

#1. $z(s) = (s^3+4s)/(s^4+7s^2+10)$ ∴ remove poles @ ∞ by continued fraction

$$s^3+4s \,\overline{)\, s^4+7s^2+10}\quad s \quad \leftarrow ay \Rightarrow \frac{\top}{\bot}$$

$$s^4+4s^2$$

$$3s^2+10 \,\overline{)\, s^3+4s}\quad \tfrac{1}{3}s \quad \leftarrow az \Rightarrow -ee-$$

$$s^3+{}^{10}\!/_3 s$$

$$\tfrac{2}{3}s \,\overline{)\, 3s^2+10}\quad \tfrac{9}{2}s \quad \leftarrow ay \Rightarrow \frac{\top}{\bot}$$

$$3s^2$$

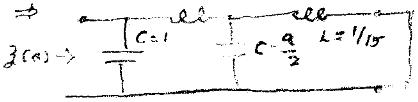
$$10 \,\overline{)\, \tfrac{2}{3}s}\quad \tfrac{2}{30}s \quad \leftarrow az \Rightarrow -ee-$$

$$\tfrac{2}{3}s$$

$$0 \leftarrow az \Rightarrow \,]$$

$$\Rightarrow z = \cfrac{1}{s + \cfrac{1}{\frac{1}{3}s + \cfrac{1}{\frac{9}{2}s + \cfrac{1}{\frac{2}{30}s = \frac{1}{15}s}}}}$$

⇒



#2. $f(s) = [(2s^2+4) + (s^3+2s)]/[(4s^2+2) + (s^3+8s)] = N(s)/D(s)$

$N(s) \Rightarrow Od/Ev = z$

$z = \frac{s(s^2+2)}{2(s^2+2)} = \frac{s}{2} \Rightarrow N(s) = (s^2+2)(s+2) \Leftarrow$ Hurwitz, not strict ~~with simple~~ 0 @ $s = \pm j\sqrt{2}$ & one in LHP @ $s = -2$

$= PR\ z$

$D(s) \Rightarrow Od/Ev = z$

$$z = \frac{s^3+8s}{4s^2+2} \Rightarrow 4s^2+2 \,\overline{)\, s^3+8s}\quad \tfrac{1}{4}s$$

$$s^3+\tfrac{1}{2}s$$

$$\tfrac{15}{2}s \,\overline{)\, 4s^2+2}\quad \tfrac{8}{15}s$$

$$4s^2$$

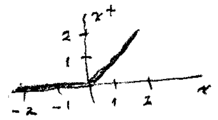
$$2 \,\overline{)\, \tfrac{15}{2}s}\quad {}^{4}\!/_{15}s$$

$$\Rightarrow z = \frac{1}{4}s + \cfrac{1}{\frac{8}{15}s + \cfrac{1}{\frac{4}{15}s}}$$

⇒ PR & $\delta[D] = 3 = \delta[z]$

⇒ Strictly Hurwitz

#3 a)



For capacitor currents

$$C_1\dot{x}_1 = \text{current down into } C_1 = G_{i_1}(x_2^+ - x_1) + i_1 - G_{c_1}x_1$$

$$C_2\dot{x}_2 = \text{'' '' '' } C_2 = G_{i_2}(x_1^+ - x_2) + i_2 - G_{c_2}x_2$$

$$\Rightarrow \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -G_{c_1}-G_{i_1} & 0 \\ 0 & -G_{c_2}-G_{i_2} \end{bmatrix} x + \begin{bmatrix} 0 & G_{i_1} \\ G_{i_2} & 0 \end{bmatrix}\begin{bmatrix} x_1^+ \\ x_2^+ \end{bmatrix} + \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Isolating $-x$ divide by $\begin{bmatrix} G_{c_1}+G_{i_1} & 0 \\ 0 & G_{c_2}+G_{i_2} \end{bmatrix}$

$$\Rightarrow \underbrace{\begin{bmatrix} \frac{C_1}{G_{c_1}+G_{i_1}} & 0 \\ 0 & \frac{C_2}{G_{c_2}+G_{i_2}} \end{bmatrix}}_{\Theta} \frac{dx}{dt} = -x + \underbrace{\begin{bmatrix} 0 & \frac{G_{i_1}}{G_{c_1}+G_{i_1}} \\ \frac{G_{i_2}}{G_{c_2}+G_{i_2}} & 0 \end{bmatrix}}_{W} x^+ + \underbrace{\begin{bmatrix} \frac{1}{G_{c_1}+G_{i_1}} & 0 \\ 0 & \frac{1}{G_{c_2}+G_{i_2}} \end{bmatrix}}_{B} i$$

b) ⟶

and x^+ = output voltage

$$v_o = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} x^+$$

b) ⟶

c) 1) when $x_i^+ = x_i > 0$ can set $x^+ = x \Rightarrow \Theta\dot{x} = (-1_2 + W)x + Bi = Ax + Bi;\ v_o = Cx \Rightarrow A = -1_2 + W$

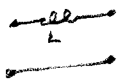
2) When $x_i^+ = 0 \Rightarrow$ can set $W = 0 \Rightarrow \Theta\dot{x} = -1_2 x + Bi = Ax + Bi;\ v_o = 0 \Rightarrow A = -1_2$

#4, $i_1 = g_1 v_y$, $i_2 = g_2 v_y$; $i_y = y v_y = -g_3(v_1 - v_2)$

$\Rightarrow v_y = \frac{-g_3}{y}(v_1 - v_2$

$\Rightarrow i_1 = \frac{-g_1 g_3}{y}(v_1 - v_2)$, $i_2 = -\frac{g_2}{y} g_3 (v_1 - v_2)$ $\Rightarrow$

a) $\Rightarrow i = Yv \Rightarrow Y = \frac{-g_3}{y(s)}\begin{bmatrix} g_1 & -g_1 \\ g_2 & -g_2 \end{bmatrix}$ if $g_2 = -g_1$ & $y = Cs$ $\Rightarrow Y = \frac{-g_1 g_3}{Cs}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow$ 

$L = -C/g_1 g_3$

$\Rightarrow L < 0$ if $\frac{C}{g_1 g_3} > 0 \Rightarrow$ if $C > 0$ then sign g_1 = + sign g_3 (= condition for $L < 0$

b) If $v_2 = 0$, $Y\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = y_{11} v_1 = i_1 \Rightarrow y_{11} = -g_1 g_3 / y(s)$

#5 $Y = \begin{bmatrix} y & -y-g \\ -y+g & y \end{bmatrix}$; if $y_{load} = G = 1/R$ then $(-G - y_{22}) v_2 = y_{21} v_1 \Rightarrow$

$\Rightarrow y_{in} = y_{11} + y_{12}\left(\frac{-1}{G + y_{22}} y_{21}\right) = \frac{\Delta_y + G y_{11}}{G + y_{22}}$; $\Delta_y = y_{11} y_{22} - y_{12} y_{21}$ = determinant Y

For $y_{in} = G = \frac{\Delta y + G y_{11}}{G + y_{22}} \Rightarrow G^2 + Gy$. $g^2 + Gy \Rightarrow G^2 = g^2$

a) $\Rightarrow$ constant R if $g = \pm G$

b) From above $v_2/v_1 = \frac{-y_{21}}{G + y_{22}} = \frac{-(-y+g)}{G + y} = \frac{y-g}{y+G} = \begin{cases} 1 & \text{if } g = -G \\ \frac{y-G}{y+G} & \text{if } g = +G \end{cases}$