Solution by Yidi Shen
Kezhen Xu

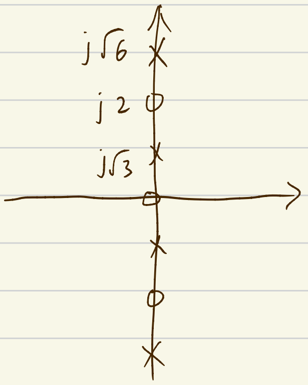
ENEE 610 HW7

#1 a) Hurwitz test

a1) $P_1(s) = s^4 + s^3 + 9s^2 + 4s + 18$

$Ev(s) = s^4 + 9s^2 + 18$

$Od(s) = s^3 + 4s$



LPR

$$\frac{Od(s)}{Ev(s)} = \frac{s^3+4s}{s^4+9s^2+18} = \frac{s(s^2+4)}{(s^2+3)(s^2+6)}$$

$$s^3+4s \,\big)\, \overset{s}{s^4+9s^2+18}$$
$$s^4+4s^2$$
$$5s^2+18 \,\big)\, \overset{\frac{1}{5}s}{s^3+4s}$$
$$s^3+\frac{18}{5}s$$
$$\frac{2}{5}s \,\big)\, \overset{\frac{25}{2}s}{5s^2+18}$$
$$5s^2$$
$$18 \,\big)\, \overset{\frac{1}{45}s}{\frac{2}{5}s}$$
$$\frac{2}{5}s$$
$$0$$

$$\frac{od(s)}{Ev(s)} = \cfrac{1}{s + \cfrac{1}{\frac{1}{5}s + \cfrac{1}{\frac{25}{2}s + \cfrac{1}{\frac{1}{45}s}}}}$$

strictly
is Hurwitz

a2) $P_2(s) = s^3 + s^2 + s + 1$ $\quad \frac{od(s)}{Ev(s)} = \frac{s^3+s}{s^2+1} = \frac{s(s^2+1)}{s^2+1} = s$

is Hurwitz

a3) $s^4 + 2s^3 + 2s^2 + 4s + 1$

$$\frac{od(s)}{Ev(s)} = \frac{2s^3+4s}{s^4+2s^2+1} = \frac{2s(s^2+2)}{(s^2+1)^2}$$

$$2s^3+4s \,\big)\, \overset{\frac{1}{2}s}{s^4+2s^2+1}$$
$$s^4+2s^2$$
$$1$$

$$\frac{Od(s)}{Ev(s)} = \frac{1}{\frac{1}{2}s + \frac{1}{2s^3+4s}} = \frac{1}{\frac{1}{2}s + \frac{1}{4s} + \frac{1}{-2s - \frac{4}{s}}}$$

not Hurwitz

a4) $P_4(s) = -P_0(s)$

$$\frac{Od_4(s)}{Ev_4(s)} = \frac{Od_0(s)}{Ev_0(s)}$$

$P_4(s)$ is Hurwitz

a5) $P_5(s) = s^5 + 3s^4 - 2s^3 + 2s^2 + 2s + 2$

negative coefficient
not Hurwitz

$$\frac{Od(s)}{Ev(s)} = \frac{s^5 - 2s^3 + 2s}{3s^4 + 2s^2 + 2}$$

$$\begin{array}{r|l} & \frac{1}{3}s \\ \hline 3s^4+2s^2+2 & s^5-2s^3+2s \\ & s^5+\frac{2}{3}s^3+\frac{2}{3}s \\ \hline & -\frac{8}{3}s^3+\frac{4}{3}s \end{array}$$

$$\begin{array}{r|l} & -\frac{9}{8}s \\ \hline -\frac{8}{3}s^3+\frac{4}{3}s & 3s^4+2s^2+2 \\ & 3s^4-\frac{3}{2}s^2 \\ \hline & \frac{7}{2}s^2+2 \end{array}$$

$$\begin{array}{r|l} & -\frac{16}{21}s \\ \hline \frac{7}{2}s^2+2 & -\frac{8}{3}s^3+\frac{4}{3}s \\ & -\frac{8}{3}s^3-\frac{32}{21}s \\ \hline & \frac{60}{21}s \end{array}$$

$$\begin{array}{r|l} & \frac{49}{40}s \\ \hline \frac{60}{21}s & \frac{7}{2}s^2+2 \\ & \frac{7}{2}s^2 \\ \hline & 2 \end{array}$$

$$\begin{array}{r|l} & \frac{30}{21}s \\ \hline 2 & \frac{60}{21}s \\ & \frac{60}{21}s \\ \hline & 0 \end{array}$$

$$\frac{Od(s)}{Ev(s)} = \frac{1}{3}s + \frac{1}{-\frac{9}{8}s + \frac{1}{-\frac{16}{21}s + \frac{1}{\frac{49}{40}s + \frac{1}{\frac{30}{21}s}}}}$$

b) $Y(s) = \frac{s^2+s+4}{s^2+s+1}$ $\quad Ev(Y) = \frac{1}{2}\left[\frac{s^2+s+4}{s^2+s+1} + \frac{s^2-s+4}{s^2-s+1}\right]$

$$= \frac{1}{2} \cdot \frac{2s^4+8s^2+8}{s^4+s^2+1}$$

$$= \frac{s^4+4s^2+4}{s^4+s^2+1}$$

zeros : $s_{1,2} = j\sqrt{2}$

$s_{3,4} = -j\sqrt{2}$

$$= \frac{(s^2+2)^2}{s^4+s^2+1}$$

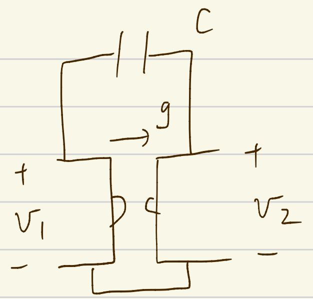
$Z(s) = \frac{s^2+s+1}{s^2+s+4}$ $\quad Ev(Z) = \frac{1}{2}\left[\frac{s^2+s+1}{s^2+s+4} + \frac{s^2-s+1}{s^2-s+4}\right]$

$$= \frac{s^4+4s+4}{s^4+7s^2+16}$$

same zeros as $Y(s)$

#2

a)



$$Y_g = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$

$$Y_c = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}$$

$$Y = \begin{bmatrix} sC & g-sC \\ -g-sC & sC \end{bmatrix}$$

let $R = 1$

$$S = (1+Y)^{-1}(1-Y)$$

$$1+Y = \begin{bmatrix} sC+1 & g-sC+1 \\ -g-sC+1 & sC+1 \end{bmatrix}$$

$$\det(1+Y) = (sC+1)^2 - (g-sC+1)(-g-sC+1)$$

$$= (sC+1)^2 - (-sC+1)^2 + g^2$$

$$= 4sC + g^2$$

$$(1+Y)^{-1} = \frac{1}{4sC+g^2}\begin{bmatrix} sC+1 & -g+sC-1 \\ g+sC-1 & sC+1 \end{bmatrix} \qquad 1-Y = \begin{bmatrix} 1-sC & 1-g+sC \\ 1+g+sC & 1-sC \end{bmatrix}$$

$$S_{11} = (1+sC)(1-sC) + (sC-g-1)(sC+g+1)$$

$$= 1 - s^2C^2 + s^2C^2 - g^2 - 2g - 1 \quad = -g^2 - 2g$$

$$S_{12} = (sC+1)(1-g+sC) + (-g+sC-1)(1-sC)$$

$$= sC - gsC + s^2C^2 + 1 - g + sC - g + gsC + sC - s^2C^2 - 1 + sC$$

$$= 4sC - 2g$$

$$S_{21} = (g+sC-1)(1-sC) + (1+g+sC)(sC+1)$$

$$= g - gsC + sC - s^2C^2 - 1 + sC + sC + 1 + gsC + g + s^2C^2 + sC$$

$$= 4sC + 2g$$

$$S_{22} = (sC+g-1)(sC-g+1) + (1+sC)(1-sC)$$

$$= s^2C^2 - g^2 + 2g - 1 + 1 - s^2C^2 \quad = -g^2 + 2g$$

$$S = \frac{1}{4sC + g^2} \begin{bmatrix} -g^2 - 2g & 4sC - 2g \\ 4sC + 2g & -g^2 + 2g \end{bmatrix}$$

b) $i = Y v$

$v^r = S v^i \Rightarrow (v - i) = S(v + i)$

$v^i = \frac{1}{2}(v + i) = \frac{1}{2}(v + Yv) = \frac{1}{2}(1 + Y)v$

$v = 2(1+Y)^{-1} v^i$

$$E \frac{dx}{dt} = Ax + Bv$$

$$= Ax + B \cdot 2(1+Y)^{-1} v^i$$

$E_s = E$

$A_s = A$

$B_s = 2(1+Y)^{-1} B$

$$\begin{cases} 2v^r = v - i \\ 2v^i = v + i \end{cases} \qquad i = v^i - v^r$$

$i = Cx \Rightarrow v^i - v^r = Cx$

$v^r = -Cx + v^i$

$C_s = -C$

$D_s = I_n$