2.) a.) $Z(s) = \dfrac{s(s^2+8)}{(s^2+4)}$

$$y(s) = \frac{s^2+4}{s(s^2+8)}$$

Choose $K = 1$

$$g = y(1) = \frac{5}{9}$$

$$C = \frac{y(1)}{1} = 5/9$$

$$\frac{y_L}{5/9} = \frac{K\,y(K) - s\,y(s)}{K\,y(s) - s\,y(K)}$$

$$= \frac{5/9 - \cancel{s}(s^2+4)/\cancel{s}(s^2+8)}{\dfrac{(s^2+4)}{s(s^2+8)} - s(5/9)}$$

$$= \frac{\dfrac{5}{9} - \dfrac{(s^2+4)}{(s^2+8)}}{\dfrac{(s^2+4)}{s(s^2+8)} - \dfrac{5s}{9}}$$

$$= \frac{s(5s^2+40 - 9s^2 - 36)}{9s^2+36 - 5s^4 - 40s^2}$$

$$= \frac{-4s^3 + 4s}{-5s^4 - 31s^2 + 36}$$

$$= \frac{4s^3 - 4s}{5s^4 + 31s^2 - 36}$$

$(s+k)(s-k)$ should be a root so,

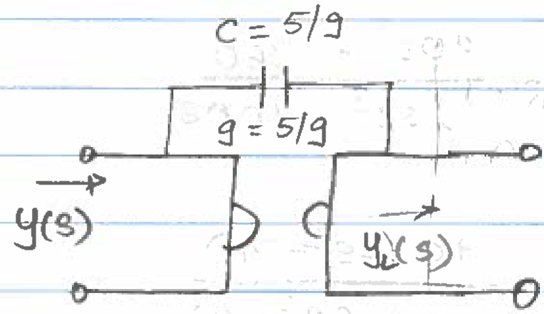
$$\begin{array}{r|l} & 4s \\ \hline s^2-1 & 4s^3 - 4s \\ & -\;4s^3 - 4s \\ \hline & 0 \end{array}
\qquad
\begin{array}{r|l} & 5s^2 + 36 \\ \hline s^2-1 & 5s^4 + 31s^2 - 36 \\ & -\;5s^4 \overset{(+)}{-} 5s^2 \\ \hline & 36s^2 - 36 \\ & -\;36s^2 \overset{(+)}{-} 36 \\ \hline & 0 \end{array}$$

$$\frac{y_L(s)}{(5/9)} = \frac{4s}{5s^2 + 36}$$

$$y_L(s) = \frac{20s/9}{5s^2+36}$$

$$= \frac{1}{s\left(\frac{9}{4}\right) + \frac{1}{s\left(\frac{5}{81}\right)}}$$

Stage 1



Now $y_L(s) = \dfrac{20s/9}{5s^2+36}$ $\longrightarrow$ Now $y_{in}(s)$

choose $K = 2$

$$g = y(2) = \frac{40/9}{56} = \frac{5}{63}$$

$$c = \frac{y(K)}{K} = \frac{5}{126}$$

$$\frac{y_L(s)}{5/63} = \frac{K'y(K) - s\,y(s)}{K\,y(s) - s\,y(K)}$$

$$= \frac{\dfrac{10}{63} - \dfrac{20s^2}{9(5s^2+36)}}{\dfrac{40s}{9(5s^2+36)} - \dfrac{5s}{63}}$$

$$= \frac{450s^2 + 3240 - 1260s^2}{2520s - 225s^3 - 1620s}$$

$$= \frac{810s^2 - 3240}{225s^3 - 900s}$$

$$= \frac{90s^2 - 360}{25s^3 - 100s}$$

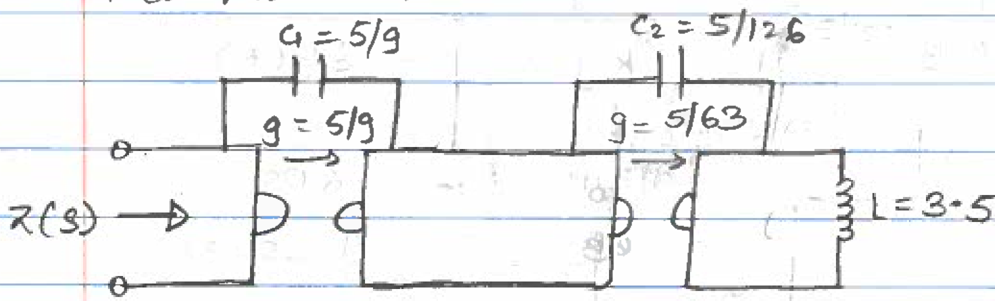
$$= \frac{90\,\cancel{(s^2-4)}}{25s\,\cancel{(s^2-4)}}$$

$$\frac{Y_L(s)}{5/63} = \frac{18}{5s}$$

$$Y_L(s) = \frac{\cancel{5}}{\cancel{63}_{7}} \times \frac{\cancel{18}^{2}}{\cancel{5}s}$$

$$Y_L(s) = \frac{1}{3.5s}$$

Realization :



b.) $x(s) = \frac{s(s^2+8)}{s^2+4}$

$y(s) = \frac{s^2+4}{s(s^2+8)}$

Now start with $k = 2$

$$y(2) = \frac{8}{24} = \frac{1}{3} = g$$

$$c = \frac{y(2)}{2} = \frac{1}{6}$$

$$\frac{y_L(s)}{1/3} = \frac{ky(k) - sy(s)}{ky(s) - sy(k)}$$

$$= \frac{2/3 - \frac{(s^2+4)}{s^2+8}}{\frac{2}{s}\frac{s^2+4}{s^2+8} - \frac{s}{3}}$$

$$= \frac{s(2s^2+16 - 3s^2-12)}{6s^2+24 - s^4 - 8s^2}$$

$$= \frac{s^3 - 4s}{s^4+2s^2-24} = \frac{s(\cancel{s^2-4})}{(s^2+6)(\cancel{s^2-4})}$$

$$y_L(s) = \frac{s/3}{s^2+6}$$

$$= \frac{s}{3s^2+18}$$

Now $K = 1$ & $y_1(s) = y_{in}(s)$

$$y(1) = \frac{1}{21} = g$$

$$\frac{y(1)}{1} = \frac{y(K)}{K} = C = \frac{1}{21}$$

$$\frac{y_L(s)}{1/21} = \left[\frac{Ky(K) - sy(s)}{Ky(s) - sy(K)}\right]$$

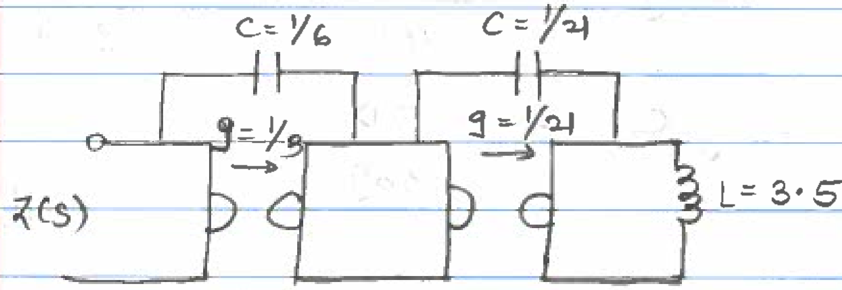
$$= \frac{\frac{1}{21} - \frac{s^2}{3s^2+18}}{\frac{s}{3s^2+18} - \frac{s}{21}}$$

$$= \frac{3s^2+18-21s^2}{21s - 3s^3-18s}$$

$$= \frac{18s^2-18}{3s^3-3s} = \frac{18}{3s}$$

$$y_L(s) = \frac{1}{21} \times \frac{18}{3s} = \frac{1}{3.5s}$$

Realization :



c.) $$Z(s) = \frac{s^3+8s}{s^2+4}$$

$$y_{in}(s) = \frac{s^2+4}{s^3+8s}$$

start with $k = 1$

$$C = \frac{y(1)}{1} = 5/9$$

$$g = y(1) = 5/9$$

From part ⓐ

$$y_L(s) = \frac{20s/9}{5s^2+36}$$

Now $k=1$ again to get new $y_L(s)$

using $y_{L_{old}} = y_{in_{new}}$

$$y(1) = g = \frac{20/9}{41} = \frac{20}{369}$$

$$\frac{y(1)}{1} = c = \frac{20}{369}$$

$$\frac{y_{L_{new}}(s)}{20/369} = \frac{ky(k) - sy(s)}{ky(s) - sy(k)}$$

$$= \frac{\frac{20}{369} - \frac{20s^2}{9(5s^2+36)}}{\frac{20s}{9(5s^2+36)} - \frac{20s}{369}}$$

$$= \frac{100s^2 + 720 - 820s^2}{820s - 100s^3 - 720s}$$

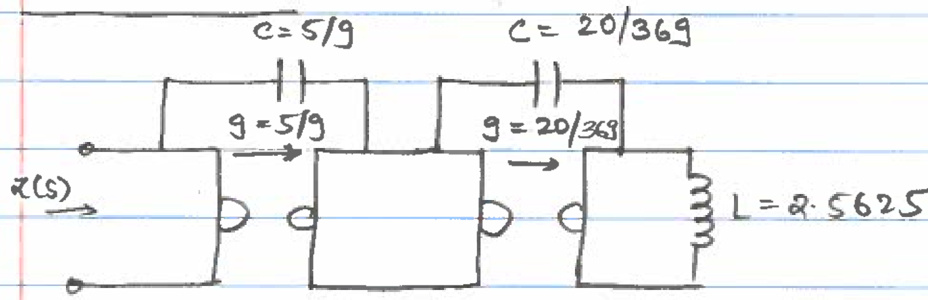
$$= \frac{720s^2 - 720}{100s^3 - 100s}$$

$$= \frac{72\cancel{0}}{10\cancel{0}s} = \frac{36}{5s}$$

$$y_{L\,new}(s) = \frac{\cancel{20}^{\,4}}{\cancel{369}_{\,41}} \times \frac{\cancel{36}^{\,4}}{\cancel{8}s}$$

$$= \frac{1}{2.5625s}$$

Realization :



For case ⓐ & ⓑ we terminate the realization with the same value of inductor (L=3.5H). However the values of gyrator and capacitor differ in these cases.

For case ⓒ since the value of "K" are 1, 1 we terminate the realization in a smaller inductor (value wise)