ENEE 610 HW 6

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① $Z(s) = \dfrac{3s^2+3s+7}{s^2+s+1}$

a) The plot of $Re[Z(j\omega)]$ is included.

$$\begin{aligned} & \quad R_{min} \quad + jX \\ Z(\sqrt{2}) &= 1.667 - 1.886j \\ &= \tfrac{5}{3} - 1.886j \end{aligned}$$

Minimum at $\omega_o = \sqrt{2}$ is $1.667 = R_{min}$

$Im[Z(j\sqrt{2})] = -1.886$

b) $Z_m(s) = Z(s) - R_{min} = \dfrac{3s^2+3s+7}{s^2+s+1} - \dfrac{5}{3}$

$$= \frac{9s^2+9s+21-5s^2-5s-5}{3(s^2+s+1)}$$

$$= \frac{4}{3}\left(\frac{s^2+s+4}{s^2+s+1}\right)$$

$Z_m(j\omega_o) = jX_o$

So, $Z_m(j\sqrt{2}) = -1.8856j \Rightarrow$ force numerator to be 0.

$$R(j\omega_o) = \frac{kZ_m(k) + \omega_o x_o}{jkx_o - \omega_o Z_m(k)}$$

$kZ_m(k) + \omega_o x_o = 0$

$k\left(\dfrac{4(k^2+k+4)}{3(k^2+k+1)}\right) + (\sqrt{2})(-1.8856) = 0$

$$\frac{4k^3+k^2+4k-8}{3(k^2+k+1)} = 0$$

$4k^3+k^2+4k-8=0$

$(4k^2+8)(k-1) = 0$

$k = 1$

$$R(s) = \frac{\left[k Z_m(k) - s Z_m(s)\right]}{\left[k Z_m(s) - s Z_m(k)\right]}$$

$$k=1, \quad R(s) = \frac{\frac{8}{3} - s\left(\frac{4(s^2+s+4)}{3(s^2+s+1)}\right)}{\left(\frac{4(s^2+s+4)}{3(s^2+s+1)} - s\left(\frac{8}{3}\right)\right)}$$

$$R(s) = \frac{2 - \frac{s^3+s^2+4s}{s^2+s+1}}{\frac{s^2+s+4}{s^2+s+1} - 2s} = \frac{2s^2+2s+2-s^3-s^2-4s}{s^2+s+4-2s^3-2s^2-2s}$$

$$= \frac{-s^3+s^2-2s+2}{-2s^3-s^2-s+4} = \frac{s^3-s^2+2s-2}{2s^3+s^2+s-4}$$

$$R(s) = \frac{(s-1)(s^2+2)}{(s-1)(2s^2+3s+4)} \approx \frac{s^2+2}{2s^2+3s+4} = \frac{1}{2+\frac{3s}{s^2+2}} = \frac{1}{2+\frac{1}{\frac{s}{3}+\frac{2}{3s}}}$$

$$Z_m(s) = \frac{1}{\frac{R(s)}{8/3} + \frac{s}{8/3}} + \frac{1}{\frac{1}{s(8/3)} + \frac{1}{(8/3)R(s)}}$$

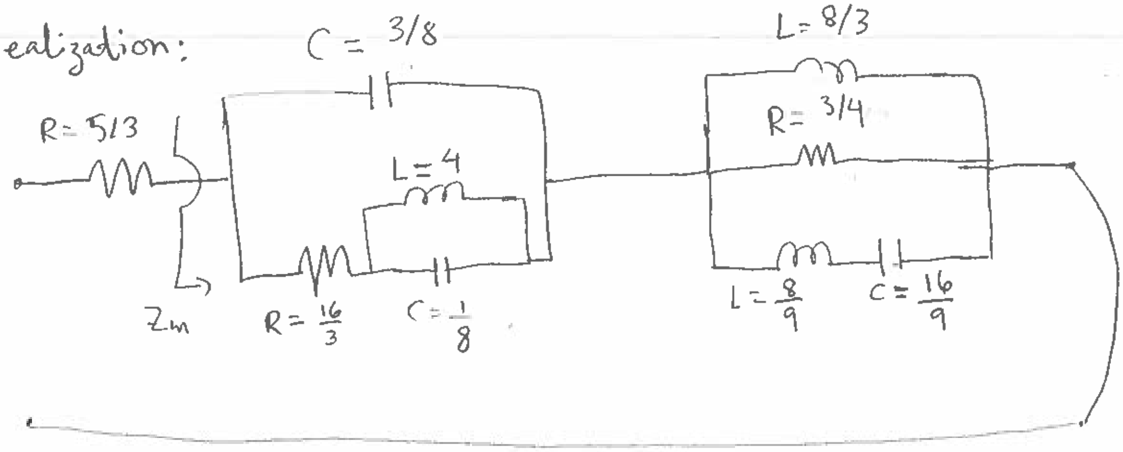
$$Z_m(s) = \frac{1}{\frac{3}{8}\left(\frac{1}{2+\frac{1}{\frac{s}{3}+\frac{2}{3s}}}\right) + \frac{s}{8/3}} + \frac{1}{\frac{1}{s(8/3)} + \frac{1}{\frac{8}{3}\left(\frac{1}{2+\frac{1}{\frac{s}{3}+\frac{2}{3s}}}\right)}}$$

$$Z_m(s) = \frac{1}{\frac{3}{16+\frac{8}{\left(\frac{s}{3}+\frac{2}{3s}\right)}} + \frac{s}{8/3}} + \frac{1}{\frac{1}{s(8/3)} + \frac{1}{\frac{8}{6+\frac{3}{\frac{s}{3}+\frac{2}{3s}}}}}$$

$$Z_m(s) = \frac{1}{\frac{1}{\frac{16}{3}+\frac{1}{\frac{s}{8}+\frac{1}{4s}}} + \frac{s}{8/3}} + \frac{1}{\frac{1}{s(8/3)} + \frac{1}{\frac{1}{\frac{3}{4}+\frac{1}{\frac{8s}{9}+\frac{16}{9s}}}}}$$

$$Z(s) = \frac{5}{3} + Z_m(s)$$

Realization:



c) $R = 3$ $\quad \delta[Z(s)] = 2$

$C = 3$

$L = 3$ $\quad$ So, $\#\ R, C, \text{or } L = \delta[Z(s)] + 1$

$Z(s)$ is PR since the function is realized using all passive components.

Problem 2

a) $Z(s) = \dfrac{s(s^2+8)}{s^2+4}$; $y(s) = \dfrac{s^2+4}{s(s^2+8)}$

$k = 1,\ z(k) = \frac{9}{5},\ y(k) = \frac{5}{9} = g$

$C = y(k)/k = 5/9$

$$\frac{y_L}{5/9} = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)} = \frac{5/9 - \left(\dfrac{s^2+4}{s^2+8}\right)}{\dfrac{s^2+4}{s(s^2+8)} - s\left(\dfrac{5}{9}\right)}$$

$$\frac{y_L}{5/9} = \frac{s(5s^2+40-9s^2-36)}{9s^2+36-5s^4-40s^2} = \frac{-4s^3+4s}{-5s^4-31s^2+36} = \frac{4s^3+4s}{5s^4+31s^2-36}$$

$(s+1)(s-1) = s^2-1$ should cancel.

So,
$$\begin{array}{r} 4s \\ s^2-1 \,\overline{\big)\, 4s^3-4s} \\ 4s^3-4s \\ \hline 0 \end{array} \qquad \begin{array}{r} 5s^2+36 \\ s^2-1 \,\overline{\big)\, 5s^4+31s^2-36} \\ 5s^4-5s^2 \\ \hline 36s^2-36 \\ 36s^2-36 \\ \hline 0 \end{array}$$

So,
$$\frac{y_L}{5/9} = \frac{4s}{5s^2+36} \Rightarrow y_L = \frac{(20s/9)}{5s^2+36}$$

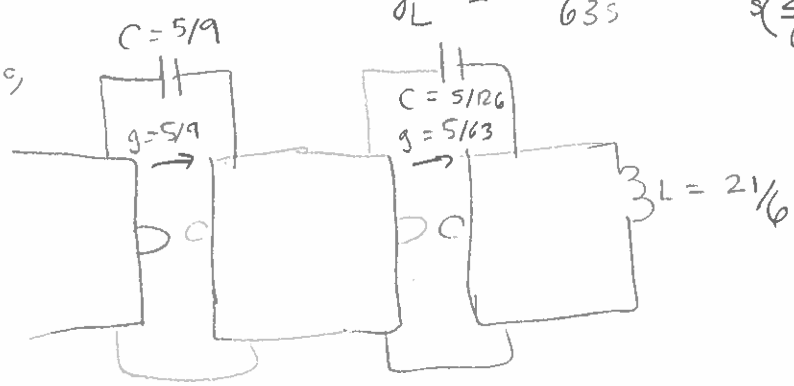
Now, $k=2 \to g = y_L(2) = \frac{5}{63}$ and $C = (5/63)/2 = \frac{5}{126}$

$$\frac{y_L}{5/63} = \left[\frac{ky(k) - sy(s)}{ky(s) - sy(k)}\right] = \frac{2(5/63) - s(20s/9)}{2\left(\frac{20s/9}{5s^2+36}\right) - s\left(\frac{5}{63}\right)} = \frac{50s^2+360-140s^2}{280s-25s^3-180s}$$

$$= \frac{18s^2-72}{5s^3+20s} = \frac{18}{5s}\left(\frac{s^2-4}{s^2-4}\right) = \frac{18}{5s} = \frac{y_L}{5/63}$$

$$y_L = \frac{18}{63s} = \frac{1}{s\left(\frac{21}{6}\right)}$$

So,



b) $k=2,\ y(2)=\frac{1}{3}=g \rightarrow c=\frac{y(k)}{k}=\frac{1}{6}$

$$\frac{y_L}{1/3}=\frac{2\left(\frac{1}{3}\right)-s\left(\frac{s^2+4}{s(s^2+8)}\right)}{2\left(\frac{s^2+4}{s(s^2+8)}\right)-s\left(\frac{1}{3}\right)}=\frac{s(2s^2+16-3s^2-12)}{6s^2+24-s^4-8s^2}=\frac{-s^3+4s}{-s^4-2s^2+24}$$

$$=\frac{s^3-4s}{s^4+2s^2-24}$$

s^2-4 is a root.

$$\begin{array}{r|l} & s \\ \hline s^2-4 & s^3-4s \\ & s^3-4s \\ \hline & 0 \end{array} \qquad \begin{array}{r|l} & s^2+6 \\ \hline s^2-4 & s^4+2s^2-24 \\ & s^4-4s^2 \\ \hline & 6s^2-24 \\ & 6s^2-24 \\ \hline & 0 \end{array}$$

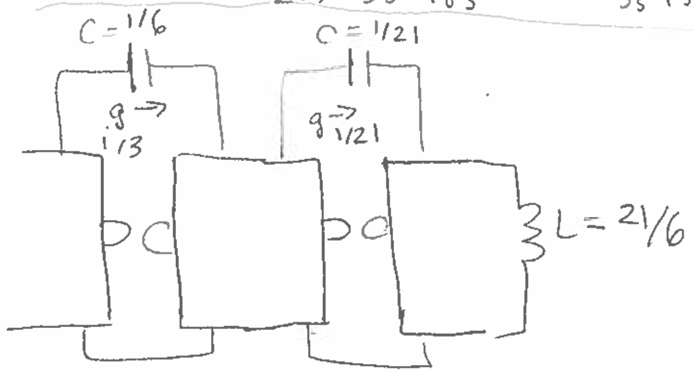
$$\frac{y_L}{1/3}=\frac{s}{s^2+6}$$

$$y_L=\frac{s}{3s^2+18}$$

$\rightarrow k=1,\ y(k)=\frac{1}{21} \rightarrow C=\frac{y(k)}{k}=\frac{1}{21}$

$$\frac{y_L}{1/21}=\frac{\frac{1}{21}-s\left(\frac{s}{3s^2+18}\right)}{\frac{s}{3s^2+18}-s\left(\frac{1}{21}\right)}$$

$$\frac{y_L}{1/21}=\frac{3s^2+18-21s^2}{21s-3s^3-18s}=\frac{-18s^2+18}{-3s^3+3s}=\frac{18}{3s}\left(\frac{s^2-1}{s^2-1}\right)=\frac{6}{s}\Rightarrow y_L=\frac{6}{21s}=\frac{1}{s\left(\frac{21}{6}\right)}$$



c) $k=1$

1st section is same as 1st section of 2a).

$$y_L(s) = \frac{(20s/9)}{5s^2+36} \;;\; k=1,\; y_L(k) = \frac{(20/9)}{5+36} = \frac{20}{369}$$

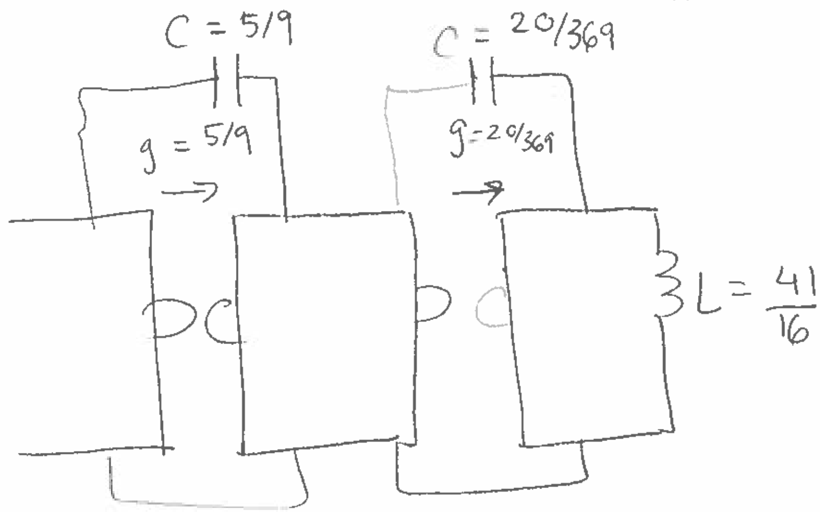
$$C = \frac{y(k)}{k} = \frac{20}{369}$$

$$\frac{y_L}{20/369} = \frac{20/369 - s\left(\frac{20s/9}{5s^2+36}\right)}{\frac{20s/9}{5s^2+36} - s\left(20/369\right)}$$

$$= \frac{100s^2 + 720 - 820s^2}{820s - 100s^3 - 720s}$$

$$= \frac{-720s^2 + 720}{-100s^3 + 100s} = \frac{-720(s^2-1)}{-100(s^2-1)} = \frac{36}{5s}$$

$$Y_L = \left(\frac{36}{5s}\right)\left(\frac{20}{369}\right) = \frac{16}{41s} = \frac{1}{s\left(\frac{41}{16}\right)}$$



Parts a and b have the same inductance, $L = 21/6$ but in part c, $L = \frac{41}{16}$ is different.

$a := 3 \qquad b := 3 \qquad c := 7 \qquad d := 1 \qquad e := 1 \qquad f := 1$

$$z(s) := \frac{\left(a \cdot s^2 + b \cdot s + c\right)}{\left(d \cdot s^2 + e \cdot s + f\right)}$$

$s1 := j \cdot \sqrt{2} \qquad z(s1) = 1.667 - 1.886j$

$\sqrt{2} = 1.414$

$R(w) := \mathrm{Re}(z(j \cdot w))$

$wmin := 0 \qquad wmax := 10 \qquad dw := 0.01$

$w := wmin, wmin + dw .. wmax$