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HW #5 PT 1

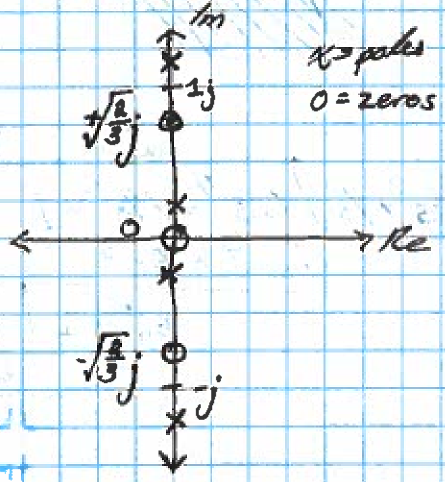
1) $Z(s) = \dfrac{s(6s^2+4)}{12s^4+14s^2+1}$ $\quad i, j = \sqrt{-1}$

a) zeros = $0, \pm\sqrt{-\frac{2}{3}}$ OR $\pm i\sqrt{\frac{2}{3}}$

Poles $\Rightarrow \dfrac{-14 \pm \sqrt{148}}{24} \Rightarrow \dfrac{-7}{12} \pm \dfrac{\sqrt{37}}{12}$

poles: $s = \pm\frac{1}{2} j \cdot \sqrt{\frac{7}{3} \pm \frac{\sqrt{37}}{3}}$

$\approx \pm .276, \pm 1.04$



b) 1ST FOSTER

$Z(s) = Z_1(s) + Z_2(s) + \ldots$

$$Z(s) = \frac{K_1}{s - j\frac{1}{2}\sqrt{\frac{7+\sqrt{37}}{3}}} + \frac{K_2}{s + j\frac{1}{2}\sqrt{\frac{7+\sqrt{37}}{3}}} + \frac{K_3}{s - j\frac{1}{2}\sqrt{\frac{7-\sqrt{37}}{3}}} + \frac{K_4}{s + j\frac{1}{2}\sqrt{\frac{7-\sqrt{37}}{3}}}$$

by PFD

$$Z(s) = K_1\left(s + j\frac{1}{2}\sqrt{\frac{7+\sqrt{37}}{3}}\right)\left(s^2 + \frac{7-\sqrt{37}}{12}\right) + K_2\left(s - j\frac{1}{2}\sqrt{\frac{7+\sqrt{37}}{3}}\right)\left(s^2 + \frac{7-\sqrt{37}}{12}\right) + K_3\left(s^2 + \frac{7+\sqrt{37}}{12}\right)\left(s + j\frac{1}{2}\sqrt{\frac{7-\sqrt{37}}{3}}\right) + K_4\left(s^2 + \frac{7+\sqrt{37}}{12}\right)\left(s - j\frac{1}{2}\sqrt{\frac{7-\sqrt{37}}{3}}\right) = .5s^3 + \frac{s}{3}$$

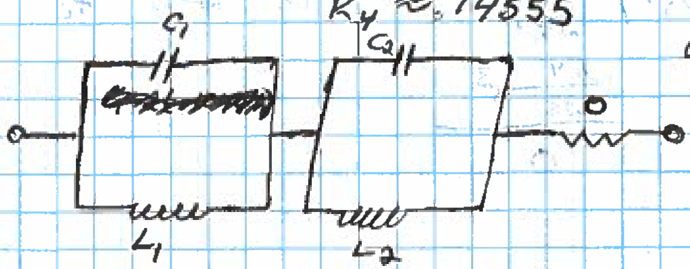
w/ $s = j\frac{1}{2}\sqrt{\frac{7+\sqrt{37}}{3}}$, $K_1 \approx 10445$ &

w/ $s = -j\frac{1}{2}\sqrt{\frac{7+\sqrt{37}}{3}}$ $K_2 \approx .10445$

K_3 .14555

$K_4 \approx .14555$

$$\Rightarrow \boxed{Z(s) = \frac{2(.10445)s}{s^2 + \frac{7+\sqrt{37}}{12}} + \frac{2(.14555)s}{s^2 + \frac{7-\sqrt{37}}{12}}}$$



Using $Z(s) = \dfrac{K_0}{s} + \sum \dfrac{2K_i s}{s^2 + \omega_i^2} + K_\infty s$

$C_i = \dfrac{1}{2K_i}$ $\quad L_i = \dfrac{2K_i}{\omega_i^2}$

$C_1 = \dfrac{1}{2(.10445\ldots)} F$ $\quad C_2 = \dfrac{1}{2(.14555\ldots)} F$

$L_1 = \dfrac{(2.10445\ldots)}{(7+\sqrt{37})/12} H$ $\quad L_2 = \dfrac{2(.14555\ldots)}{(7-\sqrt{37})/12} H$

1ST CAUER

$$\frac{s(6s^2+4)}{12s^4+14s^2+1} \rightarrow Y(s) = \frac{12s^4+14s^2+1}{6s^3+4s}$$

$2s \leftarrow ay$

$6s^3+4s \,\big)\, 12s^4+14s^2+1$

$12s^4+8s^2$

$s \leftarrow az$

$6s^2+1 \,\big)\, 6s^3+4s$

$6s^3+s$

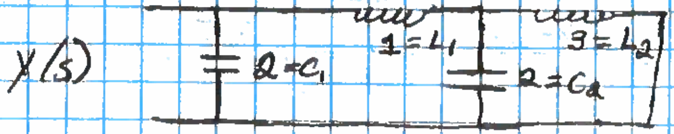
$2s \leftarrow ay$

$3s \,\big|\, 6s^2+1$

$6s^2$

$\leftarrow az$

$1 \,\big|\, 3s$



$\frac{1}{\frac{2}{3s}}$

2ND CAUER

~~$\frac{4}{4s}$ $6s^3+4s \,\big)\, 1+14s^2+12s^4$ $1+\frac{3}{2}s^2$ $12.5s^2$ $\frac{4}{12.5s}$ $6s^3+4s$~~

~~$\frac{4}{4s}$ $6s^3+4s \,\big)\, 1+14s^2+12s^4$ $1+\frac{3}{2}s^2$ $\frac{4}{12.5s}$ $12.5s^2+12s^4 \,\big)\, 6s^3+4s$~~ $\frac{96}{25}s^3+4s$

$\frac{1}{4s} \leftarrow y$

$6s^3+4s \,\big)\, 1+14s^2+12s^4$

$1+\frac{6}{4}s^2+0$

$\frac{16}{50s} \leftarrow z$

$\frac{50}{4}s^2+12s^4 \,\big)\, 4s+6s^3$

$4s+\frac{96}{25}s^3$

$\frac{625}{108s} \leftarrow y$

$\frac{54}{25}s^3 \,\big)\, \frac{50}{4}s^2+12s^4$

$\frac{25}{2}s^2+0$

$\frac{9}{50s} \leftarrow z$

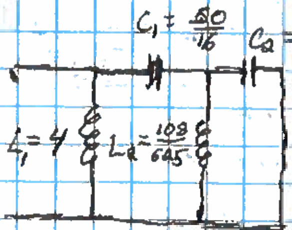
$12s^4 \,\big)\, \frac{54}{25}s^3$

$\frac{54}{25}s^3$

0

$\frac{\cancel{50}^{25}}{4} \cdot \frac{25}{\cancel{54}_{27}} = \frac{625}{108}$

$\frac{\cancel{54}^{9}}{25} \cdot \frac{1}{\cancel{12}_{2}} = \frac{9}{50}$



$L_1 = 4$ $\quad L_2 = \frac{108}{625}$

$C_1 = \frac{50}{16}$ $\quad C_2 = \frac{50}{9}$

2ND. FOSTER

$$Y(s) = \frac{12s^4+14s^2+1}{6s^3+4s} = 2s + \frac{6s^2+1}{6s^3+4s}$$

$$\frac{6s^2+1}{6s^3+4s} \Rightarrow \frac{s^2+\frac{1}{6}}{s^3+\frac{2}{3}s} = \frac{A}{s} + \frac{Bs+C}{s^2+\frac{2}{3}}$$

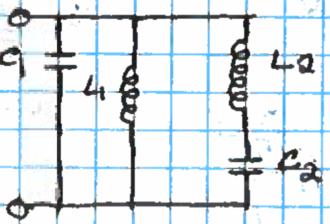
$$s^2+\frac{1}{6} = A\left(s^2+\frac{2}{3}\right) + Bs^2 + Cs$$

$$\frac{1}{6} = A\left(\frac{2}{3}\right) \qquad A = 1/4$$

$$-\frac{4}{6}+\frac{1}{6} = -\frac{1}{2} = B\left(-\frac{2}{3}\right) + C\sqrt{-\frac{2}{3}}$$

$$B = \frac{3}{4} \qquad C=0$$

$$\boxed{Y(s) = 2s + \frac{1/4}{s} + \frac{s \cdot 3/4}{s^2+2/3}}$$



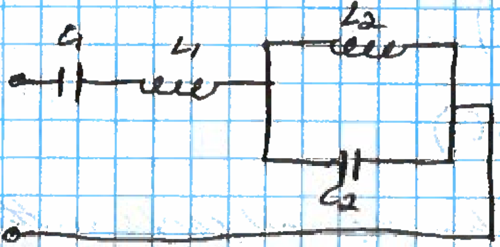
$C_1 = 2F \quad L_1 =$ 4H

$L_2 = 4/3\ H \qquad C_2 = \frac{9}{8}\ F$

c) $Y(s) = \dfrac{6s^3+4s}{12s^4+14s^2+1}$

1ST FOSTER

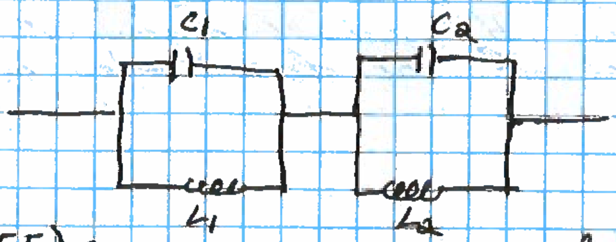
$$Z(s) = \frac{12s^4+14s^2+1}{6s^3+4s} \Rightarrow 2s + \frac{1}{4s} + \frac{\frac{3}{4}s}{s^2+2/3}$$



$C_1 = 4F \qquad L_2 = \frac{9}{8}\ H$

$L_1 = 2\ H \qquad C_2 = 4/3\ F$

2ND FOSTER



$$Y(s) = \frac{6s^3+4s}{12s^4+14s^2+1}$$

$$= \frac{2(.10445..)s}{s^2+\frac{7+\sqrt{37}}{12}} + \frac{2(.14555..)s}{s^2+\frac{7-\sqrt{37}}{12}}$$

$L_1 = \dfrac{1}{2(.10445)}\ H \qquad C_2 = \dfrac{2\times(.14555\ldots)}{(7-\sqrt{37})/12}\ F$

$C_1 = \dfrac{(2\cdot .10445\ldots)}{(7+\sqrt{37})/12}\ F \qquad L_2 = \dfrac{1}{2\cdot .14555\ldots}\ H$

1st Cauer

$$Y(s) = \frac{6s^3+4s}{12s^4+14s^2+1}$$

$$Z(s) = \frac{12s^4+14s^2+1}{6s^3+4s}$$

$$6s^3+4s \,\overline{\smash{)}\,12s^4+14s^2+1\,}\quad 2s \leftarrow Z$$

$$12s^4+8s^2$$

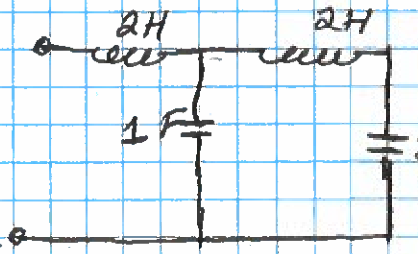
$$6s^2+1 \,\overline{\smash{)}\,6s^3+4s\,}\quad s \leftarrow Y$$

$$6s^3+s$$

$$3s \,\overline{\smash{)}\,6s^2+1\,}\quad 2s \leftarrow Z$$

$$6s^2+0$$

$$1 \,\overline{\smash{)}\,3s\,}\quad 3s \leftarrow Y$$



2nd Cauer

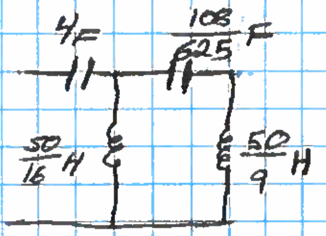
$$4s+6s^3 \,\overline{\smash{)}\,1+14s^2+12s^4\,}\quad \frac{1}{4s} \leftarrow Z$$

$$\hookrightarrow \frac{16}{50s} \leftarrow Y$$

$$\hookrightarrow \frac{625}{108s} \leftarrow Z$$

$$\hookrightarrow \frac{9}{50s} \leftarrow Y$$

FROM PTB



d) $Z_p = \left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)^{-1} \Rightarrow \left(\frac{12s^4+14s^2+1}{6s^3+4s}+1\right)^{-1}$

$$\Rightarrow \left(\frac{12s^4+6s^3+14s^2+4s+1}{6s^3+4s}\right)^{-1}$$

$$\Rightarrow \frac{6s^3+4s}{12s^4+6s^3+14s^2+4s+1} = Z_p(s)$$

This is Hurwitz polynomial because the denominator has all roots in the negative LHP and the degree of the numerator does not excee that of the denominator by more than one.