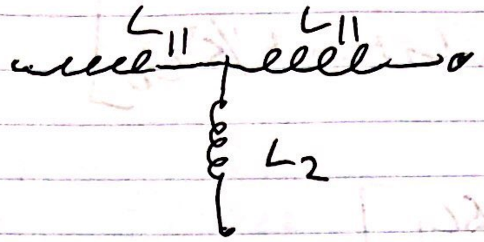
HW 3 solutions #2

a.



$$V_1 = sL_1 I_1 + sL_2(I_1 + I_2)$$

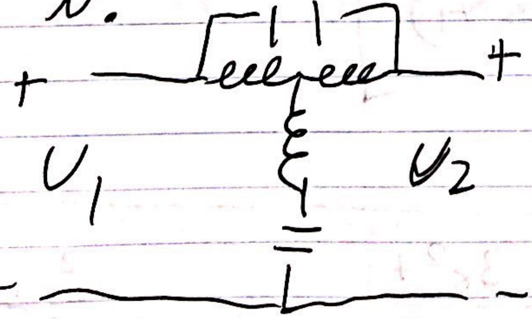
$$V_2 = sL_1 I_2 + sL_2(I_2 + I_1)$$

$$Z = s\begin{bmatrix} L_1 + L_2 & L_2 \\ L_2 & L_1 + L_2 \end{bmatrix}$$

$$L_{21} = L_{12} = L_2 = M$$

$$L_{11} = L_{22} = L_1 + L_2$$

b.



Z lallen = $V_1 = I_1 sL_1 + sL_2(I_1 + I_2) + 1/sC_2(I_1 + I_2)$

$V_2 = I_2 sL_2 + sL_2(I_1 + I_2) + 1/sC_2(I_1 + I_2)$

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$$Z = s\begin{bmatrix} L_1+L_2+1/s^2C_2 & L_2+1/s^2C_2 \\ L_2+1/s^2C_2 & L_1+L_2+1/s^2C_2 \end{bmatrix}$$

$$Y_C = \begin{bmatrix} sC_1 & -sC_1 \\ -sC_1 & sC_1 \end{bmatrix}$$

$$Z^{-1} = \begin{bmatrix} \frac{s^2L_1C_2+s^2L_2C_2+1}{s(s^2L_1^2C_2+2s^2L_1L_2C_2+2L_1)} & \frac{-s^2C_2L_2+1}{s(s^2L_1^2C_2+2s^2L_1L_2C_2+2L_1)} \\ \frac{-s^2C_2L_2+1}{s(s^2L_1^2C_2+2s^2L_1L_2C_1+2L_1)} & \frac{s^2L_1C_1+s^2L_1L_2+1}{s(s^2L_1^2C_2+2s^2L_1L_2C_2+2L_1)} \end{bmatrix}$$

5/ram problem

$L_1 = 2,\ L_2 = -3/4,\ C_2 = 4,\ C_1 = 1/4$

$$Z^{-1} = \begin{bmatrix} \frac{5s^2+1}{4s(s^2+1)} & \frac{-3s^2+1}{4s(s^2+1)} \\ \frac{-3s^2+1}{4s(s^2+1)} & \frac{5s^2+1}{4s(s^2+1)} \end{bmatrix}$$

$$Y = \frac{1}{4}s\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$Y_T = Z^{-1} + Y_C = \begin{bmatrix} \frac{3s^2+1}{2s(s^2+1)} & \frac{s^2-1}{2s(s^2+1)} \\ \frac{s^2-1}{2s(s^2+1)} & \frac{3s^2+1}{2s(s^2+1)} \end{bmatrix}$$

c. $-G V_2 = Y_{21} V_1 + Y_{22} V_2$

$(-G - Y_{22}) V_2 = Y_{21} V_1$

$\frac{V_2}{V_1} = \frac{Y_{21}}{-G - Y_{22}}$

$= \frac{-Y_{21}}{G + Y_{22}}$

$= \frac{\frac{s^2-1}{2s(s^2+1)}}{G + \frac{3s^2+1}{2s(s^2+1)}}$

$\boxed{= \frac{s^2-1}{G(2s(s^2+1)) + 3s^2+1}} = V_2/V_1$

If $G = 0$

$= \frac{s^2-1}{3s^2+1}$ zeroes at $s = 1, -1$
poles at $s = \frac{1}{\sqrt{3}} j, \frac{-1}{\sqrt{3}} j$

d

$-G_2 V_2 = y_{21} V_1 + y_{22} V_2$

$V_2 = \frac{-(s^2-1)}{G(2s(s^2+1)) + 3s^2+1}$

$I_1 = y_{11} V_1 + y_{12} V_2$

$y_{in} = y_{11} + y_{12} \frac{V_2}{V_1}$

$$\boxed{Z_{in} = \frac{3s^2+1}{2s(s^2+1)} - \frac{(s^2-1)^2}{6(2s(s^2+1)+3s^2+1)}}$$