ENEE 610

HW 3

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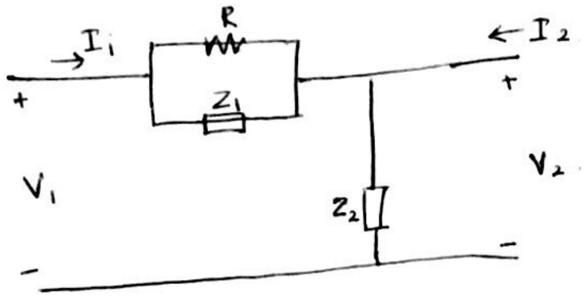
① a)



$$V_1 = i_1\left(\frac{RZ_1}{R+Z_1}\right) + (i_1+i_2)Z_2 \quad — ①$$

$$V_2 = (i_1+i_2)Z_2 \quad — ②$$

$$\therefore \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{RZ_1}{R+Z_1} + Z_2 & Z_2 \\ Z_2 & Z_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\therefore Z_{2\,port} = \begin{bmatrix} \frac{RZ_1}{R+Z_1} + Z_2 & Z_2 \\ Z_2 & Z_2 \end{bmatrix}$$

b) $R_{load} = R$

$\therefore -V_2 = i_2 R$

② ⇒ $V_2 = (i_1+i_2)Z_2$

$\therefore -i_2 R = (i_1+i_2)Z_2$

$\therefore Z_2 i_1 = -i_2(R+Z_2)$.

$$Z_{in} = \frac{V_1}{i_1} = \frac{-\frac{i_2}{Z_2}\left(\frac{RZ_1}{R+Z_1}\right)(R+Z_2) \cancel{i_2 R} - i_2 R}{-i_2\frac{(R+Z_2)}{Z_2}}$$

$$= \frac{R^2 Z_1 + RZ_1Z_2 + R^2 Z_2 + RZ_1Z_2}{(R+Z_2)(R+Z_1)} = \frac{RZ_1}{R+Z_1} + \frac{RZ_2}{R+Z_2}.$$

c). $Z_{in} = R$ when $\dfrac{Z_1}{R} = \dfrac{R}{Z_2}$.

$$Z_{in} = \frac{RZ_1}{R+Z_1} + \frac{RZ_2}{R+Z_2} = R.$$

$$\cancel{RZ_1} + \cancel{Z_1Z_2} + \cancel{RZ_2} + Z_1Z_2 = R^2 + \cancel{RZ_2} + RZ_1 + \cancel{Z_1Z_2}.$$

$$R^2 = Z_1Z_2.$$

d) $$\frac{V_2}{V_1} = \frac{-i_2R}{-\frac{i_2}{Z_2}\left(\frac{RZ_1}{R+Z_1}\right)(R+Z_2) - i_2R} = \frac{1}{\frac{Z_1}{Z_2}\left(\frac{R+Z_2}{R+Z_1}\right)+1}$$

$$= \frac{RZ_2 + Z_1Z_2}{RZ_1 + Z_1Z_2 + RZ_2 + Z_1Z_2} = \frac{RZ_2 + R^2}{RZ_1 + 2R^2 + RZ_2.} = \frac{R+Z_2}{Z_1+Z_2+2R.}$$

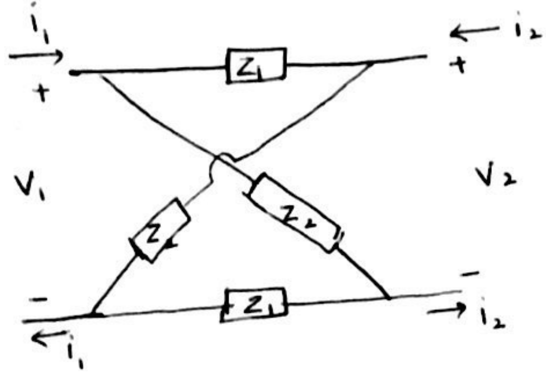
$$\left[Z_1 = \frac{R^2}{Z_2}\right] \quad \therefore \frac{V_2}{V_1} = \frac{R + \cancel{R}Z_2}{\frac{R^2}{Z_2} + Z_2 + 2R} = \frac{Z_2(R+Z_2)}{R^2 + Z_2^2 + 2RZ_2}$$

$$= \frac{Z_2}{R+Z_2}$$

when $Z_2 = LS$ $$\frac{V_2}{V_1} = \frac{LS}{R+LS}. = \frac{S}{S + R/L}.$$

b circuit

(a)



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{i_2=0} = \frac{Z_1+Z_2}{2} = Z_{22}.$$

$$Z_{12} = Z_{21} = \left.\frac{V_1}{I_2}\right|_{i_1=0} = \frac{Z_2-Z_1}{2}.$$

$$\therefore \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{Z_1+Z_2}{2} & \frac{Z_2-Z_1}{2} \\ \frac{Z_2-Z_1}{2} & \frac{Z_1+Z_2}{2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}.$$

$$\therefore Z_{2\,port} = \begin{bmatrix} \frac{Z_1+Z_2}{2} & \frac{Z_2-Z_1}{2} \\ \frac{Z_2-Z_1}{2} & \frac{Z_1+Z_2}{2} \end{bmatrix}$$

$$V_2 = \left(\frac{Z_2-Z_1}{2}\right) i_1 + \left(\frac{Z_1+Z_2}{2}\right) i_2 = -i_2 R.$$

$$(Z_2-Z_1)\, i_1 = -i_2(Z_1+Z_2+2R)$$

(b) $R_{load} = R$

$\Rightarrow V_2 = -i_2 R$

$$Z_{in} = \frac{V_1}{i_1} = \frac{i_1\left(\frac{Z_1+Z_2}{2}\right) + \frac{(Z_2-Z_1)}{2}\left[\frac{-(Z_2-Z_1)}{Z_1+Z_2+2R}\right] i_1}{i_1}$$

$$= \frac{Z_1+Z_2}{2} - \frac{(Z_2-Z_1)^2}{(Z_1+Z_2+2R)2} = \frac{(Z_1+Z_2)^2 + 2RZ_1 + 2RZ_2 - (Z_2-Z_1)^2}{(Z_1+Z_2+2R)2}$$

$$= \frac{4Z_1Z_2 + 2R(Z_1+Z_2)}{(Z_1+Z_2+2R)2} = \frac{R(Z_1+Z_2)+2Z_1Z_2}{Z_1+Z_2+2R}$$

(c) . $Z_{in} = R$ when $\dfrac{Z_1}{R} = \dfrac{R}{Z_2}$.

$$Z_{in} = \frac{R(Z_1+Z_2) + 2Z_1Z_2}{Z_1+Z_2+2R} = R \; . \quad \Rightarrow \quad Z_1Z_2 = R^2 .$$

(d) .
$$\frac{V_2}{V_1} = \frac{-i_2 R}{\left(\frac{Z_1+Z_2}{2}\right)\left(-i_2 \frac{(Z_1+Z_2+2R)}{Z_2-Z_1}\right) + i_2\left(\frac{Z_2-Z_1}{2}\right)},$$

$$= \frac{2R(Z_2-Z_1)}{(Z_1+Z_2)^2 + (Z_1+Z_2)2R - (Z_2-Z_1)^2} = \frac{2R(Z_2-Z_1)}{4Z_1Z_2 + 2R(Z_1+Z_2)} .$$

$$= \frac{R\left(Z_2 - \frac{R^2}{Z_2}\right)}{2R^2 + 2R\left(Z_2 + \frac{R^2}{Z_2}\right)} = \frac{Z_2^2 - R^2}{Z_2^2 + R^2 + 2RZ_2} . = \frac{Z_2 - R}{Z_2 + R} .$$

$$\left[Z_1 = \frac{R^2}{Z_2}\right]$$

$$Z_2 = LS \quad \Rightarrow \quad \frac{V_2}{V_1} = \frac{LS - R}{LS + R} = \frac{S - R/L}{S + R/L} .$$