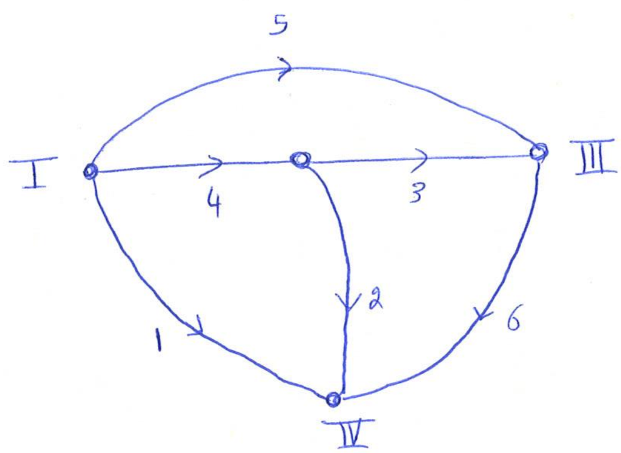
ENEE 610 Fall H.W. 1 Due Tu 09/10

ShaybanGhabech #1

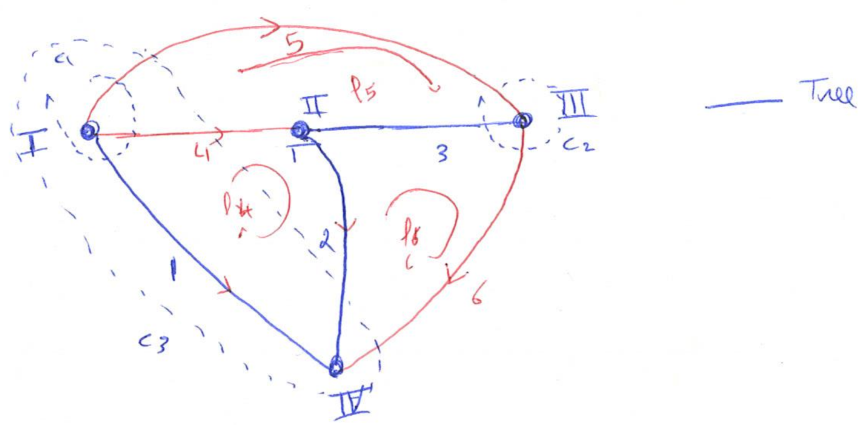
Solution:

#1

a) The corresponding graph for the given circuit is:



b) For the cutset and tie set matrices we use the following KCL and KVLs:



KCL at cutset 1:

$$i_1 + 0i_2 + 0i_3 + i_4 + i_5 + 0i_6 = 0$$

KCL at cutset 2:

$$0i_1 + i_2 + 0i_3 - i_4 - i_5 + 0i_6 = 0$$

KCL at cut set 3:

$$0i_1 + 0i_2 + i_3 + 0i_4 + i_5 - i_6 = 0$$

Placing these in matrix format we get:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}}_{C} \times \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

C: cut set matrix.

Using KVLs we can obtain T set matrix:

KVL in loop 4:

$$-V_1 + V_2 + 0V_3 + V_4 + 0V_5 + 0V_6 = 0$$

$$-V_1 + V_2 + V_3 + 0V_4 + V_5 + 0V_6 = 0$$

$$0V_1 - V_2 + V_3 + 0V_4 + 0V_5 + V_6 = 0$$

Arranging the equation in matrix format we obtain;

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}}_{\text{Tiset matrix}} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

c) For the augmented incidence matrix we have:

$$A_a = \begin{array}{c} \\ \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV} \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \\ -1 & -1 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

If we delete last row:

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix}$$

To determine how many trees we need to calculate $\det(AA^T)$

$$(A \times A^T) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$\det(A A^T) = 3(9-1) + 1(-3-1) - 1(1+3)$

$= 24 - 4 - 4 = 16$

# of trees this circuit has is $\boxed{16}$.

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Question 02

a) $e = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad j = \underline{0}_6$ $\quad G_3 = \frac{1}{R_3}, \ G_4 = \frac{1}{R_4}, \ G_6 = \frac{1}{R_6}$

Equations:

$v = Lsi$

$i = Csv$

Now to find $A(s)$ and $B(s)$ such that $A(s)\,v = B(s)\,i$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & sC_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6 \end{bmatrix}}_{A(s)} v = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & sL_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{B(s)} i$$

$v_b = v + e$, $i_b = i + j$

$v = v_b - e$, $i = i_b - j$

$v = C^T v_t - e$, $i = J^T i_\ell - j$

b) $A(s)v = B(s)i \longrightarrow A(s)\left[C^T v_t - e\right] = B(s)\left[J^T i_\ell - j\right]$

$\Rightarrow \begin{bmatrix} AC^T & -BJ^T \end{bmatrix} \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = Ae - Bj$

$Ae = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $Bj = \underline{0}_6 \Rightarrow Ae - Bj = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow$ (A)

Now we need to find $A\mathcal{C}^T$ and $B\mathcal{J}^T$

$$A\mathcal{C}^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & sC_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & sC_2 & 0 \\ 0 & 0 & G_3 \\ G_4 & -G_4 & 0 \\ 1 & -1 & 1 \\ 0 & G_6 & -G_6 \end{bmatrix} \rightarrow \text{(B)}$$

$$B\mathcal{J}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & sL_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & sL_5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{(C)}$$

Circuit equations are then obtained using (A), (B) and (C)

$$\begin{bmatrix} A\mathcal{C}^T & -B\mathcal{J}^T \end{bmatrix} \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = Ae - Bj$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & sC_2 & 0 \\ 0 & 0 & G_3 \\ G_4 & -G_4 & 0 \\ 1 & -1 & 1 \\ 0 & G_6 & -G_6 \end{bmatrix}}_{A\mathcal{C}^T}\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 0 & -sL_5 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{-B\mathcal{J}^T} \begin{bmatrix} v_{1_t} \\ v_{2_t} \\ v_{3_t} \\ i_{4_\ell} \\ i_{5_\ell} \\ i_{6_\ell} \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$