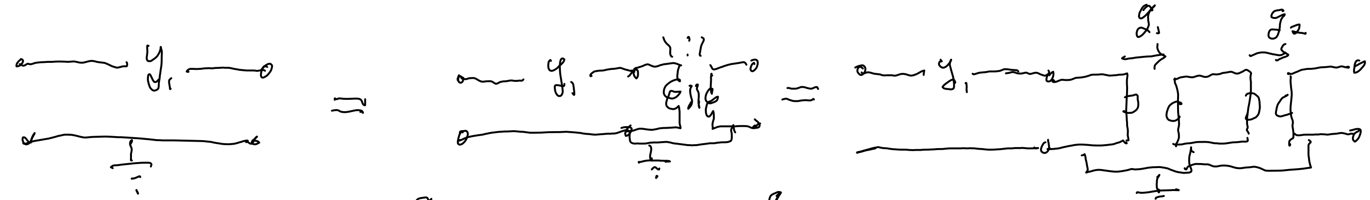
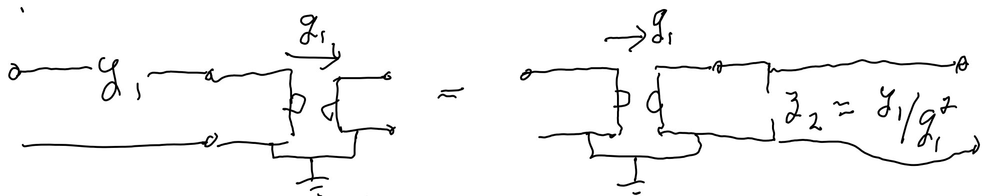
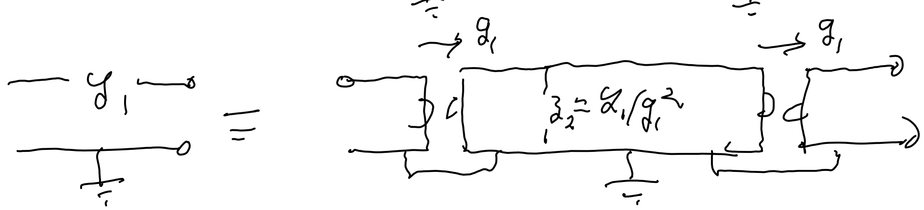
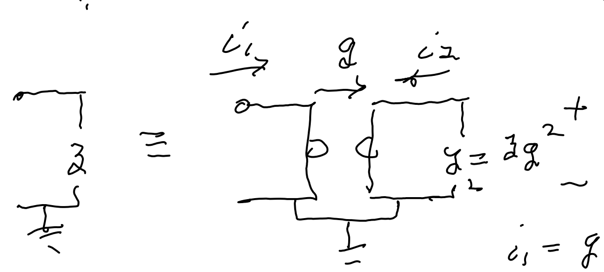
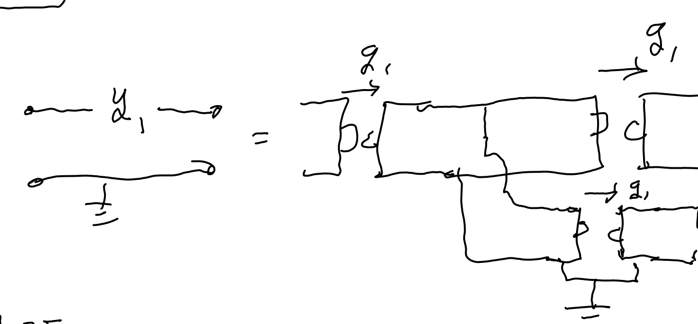


but









$y_2 = \dfrac{g_1^2 y_1}{g_1^2} = y_1$

$i_1 = g\, v_2$

$i_2 = -\cancel{g}\, v_1 = -y_L v_2 = -$

$v_1 = \frac{y_L}{g} v_2 = z\, i_1 = z g\, v_2 \Rightarrow y_2 = g^2 z$

oscillators

Linear time invariant

$$\ddot{x} + \omega_0^2 x = 0 , \quad x(0), \dot{x}(0) \text{ given}$$

$$\frac{\ddot{x}}{\omega_0^2} + x = \frac{d^2 x}{\omega_0^2 dt^2} + x = \frac{d^2 x}{d\tau^2} + x = 0$$

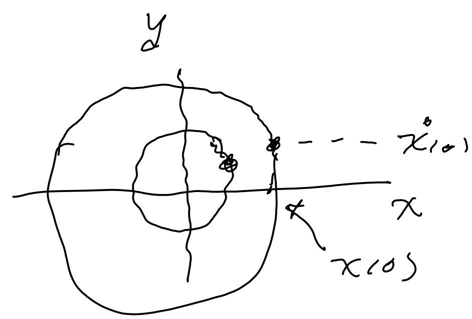
$$\tau = \omega_0 t$$

$$\dot{} = \frac{d}{d\tau}$$

$$\dot{x} = y$$
$$\dot{y} = \ddot{x} = -x ,$$

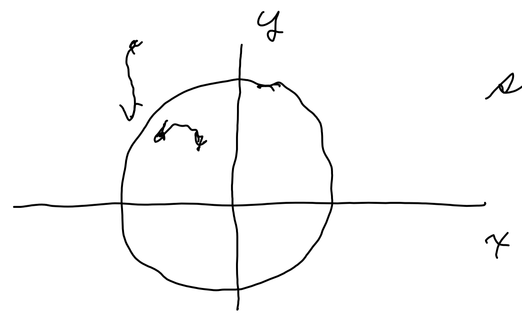
$$L = x^2 + y^2 \geq 0 , \quad \frac{dL}{d\tau} = 2x\dot{x} + 2y\dot{y} = 2xy + 2y(-x) = 0$$

$$L = \text{constant} = L(0) = x(0)^2 + \dot{x}(0)^2 =$$



Van der Pol oscillator

$$\ddot{x} + \epsilon(x^2-1)\dot{x} + \omega_0^2 x = 0 \quad , \; x(0), \dot{x}(0) \text{ given}, \; \epsilon > 0 \text{ is a parameter}$$



structurally stable

$$\underbrace{\frac{d}{dt}\left(\dot{x} + \epsilon\left(\frac{x^3}{3} - x\right)\right)}_{\frac{dy}{dt}} + \omega_0^2 x = 0$$

$$\dot{y} = -\omega_0^2 x \quad , \quad y = \dot{x} + \epsilon\left(\frac{x^3}{3} - x\right)$$

$$\dot{x} = y - \epsilon\left(\frac{x^3}{3} - x\right)$$

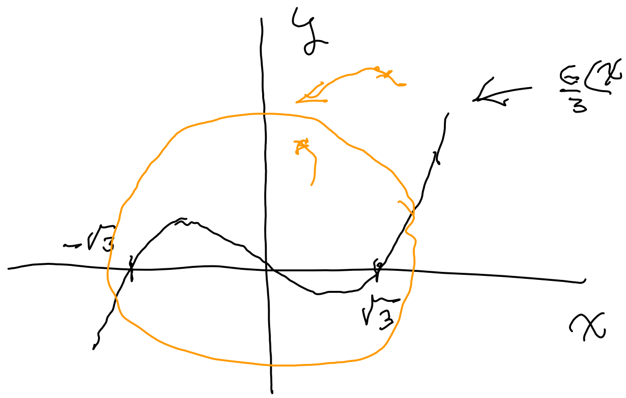
$$L = (\omega_0^2 x)^2 + y^2 \; ; \quad \frac{dL}{dt} = 2\omega_0^2 x\dot{x} + 2y\dot{y} = 2\left(\omega_0^2 x\left[y - \epsilon\left(\frac{x^3}{3} - x\right)\right] + y(-\omega_0^2 x)\right)$$

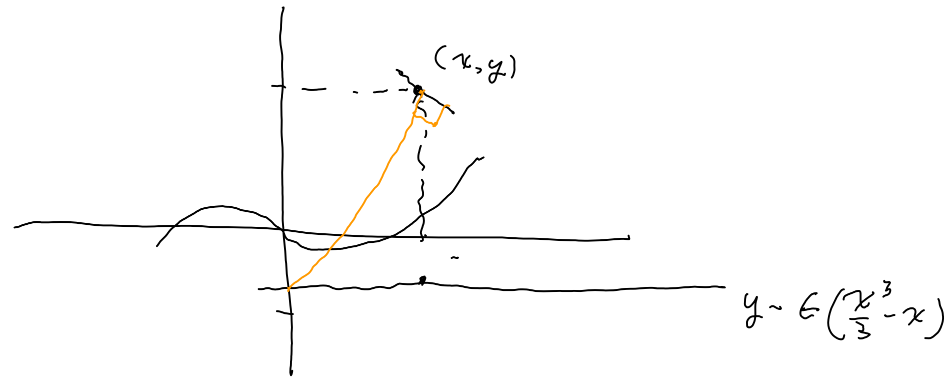
$$= -2\epsilon\left(\frac{x^3}{3} - x\right) \qquad \text{transition @ } \frac{x^3}{3} - x = 0$$

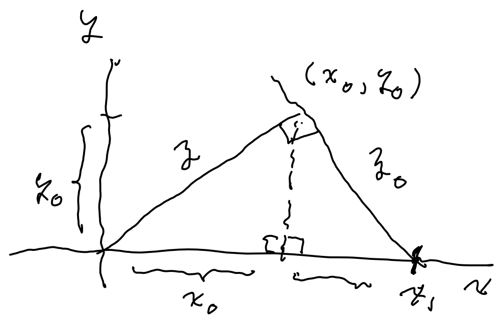
$$x^2 = 3 \text{ @ } x = \pm\sqrt{3}$$

to find limit cycle

$$\frac{\epsilon}{3}(x^3-3x)=y \qquad \frac{dy/dt}{dx/dt}=\frac{dy}{dx}=\frac{-\omega_0^2 x}{y-\epsilon\left(\frac{x^3}{3}-x\right)}$$







$$z^2 = x_0^2 + y_0^2 \quad , \quad z_0^2 = y_0^2 + (x_1 - x_0)^2 \quad ; \quad x_1^2 = z^2 + z_0^2 = y_0^2 + (x_1 - x_0)^2 + x_0^2 + y_0^2$$

$$x_1^2 = 2y_0^2 + x_1^2 - 2x_1x_0 + x_0^2 + x_0^2$$

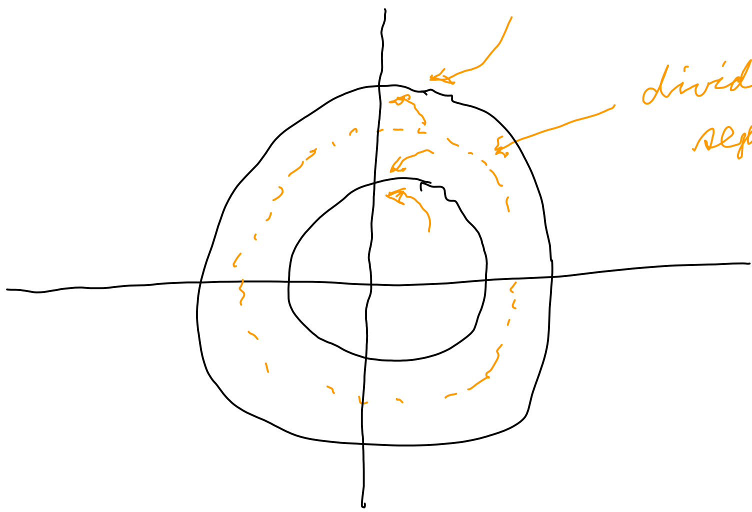
$$2x_1x_0 = 2y_0^2 + 2x_0^2$$

$$\Leftarrow x_1 = x_0 + y_0^2/x_0 \Rightarrow x_1 - x_0 = \frac{y_0^2}{x_0}$$

line $z \Rightarrow$ slope $= y_0/x_0$ slope of z_0 line $= \frac{-y_0}{x_1 - x_0}$

$$= \frac{-y_0}{y_0^2/x_0} = -\frac{x_0}{y_0}$$

inverse



$$\ddot{x} + \underset{\text{constant}}{\underset{\shortparallel}{a}}\dot{x} + x = 0 \qquad x = b(t)\, y$$

$$\dot{x} = \dot{b}y + b\dot{y}, \qquad \ddot{x} = \ddot{b}y + \dot{b}\dot{y} + \dot{b}\dot{y} + b\ddot{y}$$

$$\ddot{b}y + 2\dot{b}\dot{y} + b\ddot{y} + a(\dot{b}y + b\dot{y}) + by = 0$$

$$b\ddot{y} + (2\dot{b} + ab)\dot{y} + (\ddot{b} + a\dot{b} + b)y = 0$$

$$\ddot{y} + \left(\underbrace{\ddot{b} + a\dot{b} + b}_{b}\right) y = 0$$

set ~~coef~~ $\dot{y} \Rightarrow 0$

$$\dot{b} + \frac{a}{2}b = 0 \Rightarrow b(0)e^{-\frac{a}{2}t}1(t) = b(t)$$

$$\ddot{b} = -\frac{a}{2}\dot{b}$$

$$\frac{-\frac{a}{2}\dot{b} + a\dot{b} + b}{b} = \frac{+\frac{a}{2}\left(-\frac{a}{2}b\right) + b}{b} = -\frac{a^2}{4} + 1 \Longrightarrow \ddot{y} + \left(1 - \frac{a^2}{4}\right)y = 0$$

$$x = b(t), y(t)$$