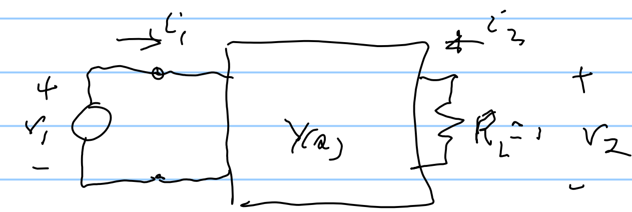
Note Title 10/29/2019

ENEE 610
10/31/19

$$\frac{V_2}{V_1}(s) =$$



$$i_1 = y_{11} v_1 + y_{12} v_2, \qquad i_2 = y_{21} v_1 + y_{22} v_2 = -G_L \cdot v_2$$

$$y_{21} v_1 = -(G_L + y_{22}) v_2$$

$$\Rightarrow \frac{v_2}{v_1} = \frac{-y_{21}}{G_L + y_{22}} = \frac{-y_{21}/G_L}{1 + y_{22}/G_L} \quad \text{let } G_L = 1 \Longrightarrow \frac{V_2}{V_1}(s) = \frac{-y_{21}}{1 + y_{22}} = \frac{N(s)}{D(s)}$$

If $D(s)$ is strictly Hurwitz (true for $B_n(s)$) then

$$D(s) = \mathrm{Ev}\, D(s) + \mathrm{Od}\, D(s) = \mathrm{Ev}\, D(s)\left[1 + \frac{\mathrm{Od}\, D(s)}{\mathrm{Ev}\, D(s)}\right] = \mathrm{Od}\, D(s)\left[1 + \frac{\mathrm{Ev}\, D(s)}{\mathrm{Od}\, D(s)}\right]$$

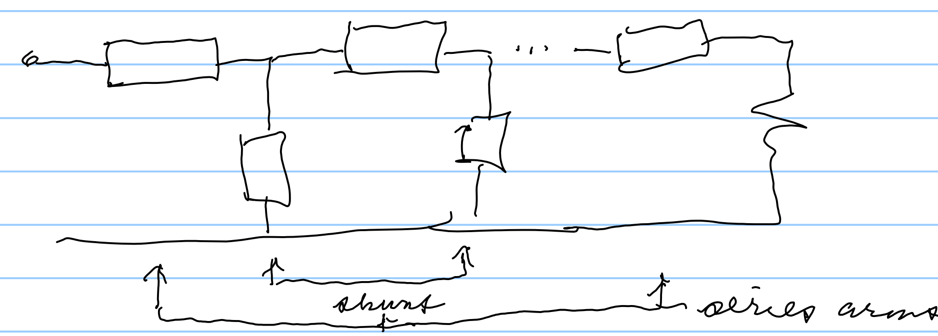
$$\frac{V_2}{V_1}(s) = \frac{-y_{21}}{1+y_{22}} = \frac{N(s)/Ev\,D(s)}{1 + Od\,D(s)/Ev\,(D(s))} \Longrightarrow \text{choose } y_{22} = \frac{Od\,D(s)}{Ev\,(D(s))}$$

Ex: $$\frac{V_2}{V_1}(s) = \frac{k}{(s+1)^3} = \frac{k}{s^3+3s^2+3s+1} = \overset{\text{no LPR}}{\frac{k}{(3s^2+1)+(s^3+3s)}} = \frac{k/(3s^2+1)}{1+\frac{(s^3+3s)}{3s^2+1}}$$

$\Uparrow$ 3 zeros of transmission @ ∞

$$\therefore\ y_{22} = \frac{s(s^2+3)}{3s^2+1} = \frac{s(s^2+3)}{3(s^2+1/3)}$$

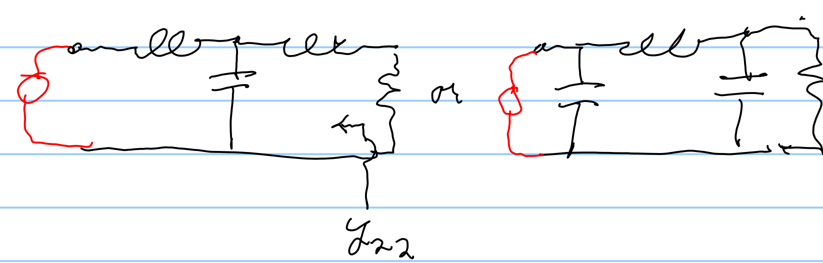
Look at ladder circuits



where series arms are open $\Rightarrow$ a zero of transmission
shunt " " short " " "

if all zeroes are at ∞, series arms have $z(\infty)=\infty \Rightarrow z(s)=sL$
shunt " " $y(\infty)=\infty \Rightarrow y(s)=1/sC$

In our Ex: 3 zeroes



synthesize y_{22} by 1st Cauer
$\underbrace{\text{shows a pole @ } \infty}$ of $\hat{z}_{22} = 1/y_{22}$

∴ need the possible y_{22} as our choice above has $y_{22}\Big|_{s\to\infty} = \frac{s^3}{3s^2}$ is a pole @ ∞

$\therefore$ choose for $\dfrac{V_2}{V_1} = \dfrac{k/\text{Od}\,D(s)}{1 + \dfrac{\text{Ev}\,D(s)}{\text{Od}\,D(s)}} \Rightarrow y_{22} = \dfrac{\text{Ev}\,D(s)}{\text{Od}\,D(s)} = \dfrac{3s^2+1}{s^3+3s}$

synthesize y_{22} by 1st Cauer: $y_{22} = \dfrac{1}{\dfrac{s^3+3s}{3s^2+1}} = \dfrac{1}{\frac{1}{3}s + \dfrac{1}{\frac{9}{8}s + \dfrac{1}{\frac{8}{3}s + 0}}}$

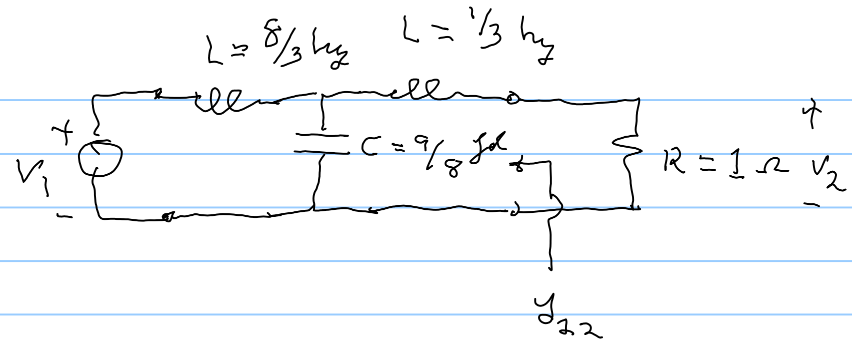
Z Y Z

Y

Z

$Z \longrightarrow$

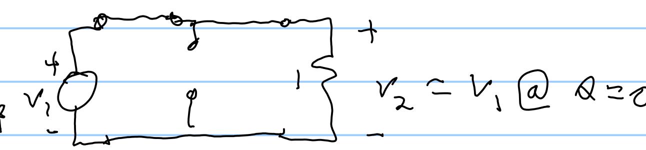
$$
\begin{array}{r|l}
 & \tfrac{1}{3}s \\
3s^2+1 & s^3 + 3s \\
 & s^3 + \tfrac{1}{3}s \\
\hline
 & \tfrac{8}{3}s
\end{array}
\qquad
\begin{array}{r|l}
 & \tfrac{9}{8}s \\
\tfrac{8}{3}s & 3s^2+1 \\
 & 3s^2 \\
\hline
 & 1
\end{array}
\qquad
\begin{array}{r|l}
 & \tfrac{8}{3}s \\
1 & \tfrac{8}{3}s
\end{array}
$$



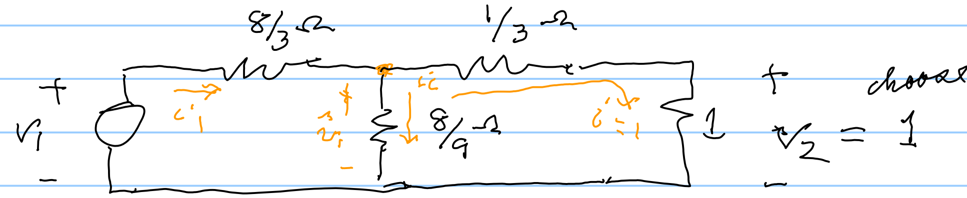
$$\frac{V_2}{V_1} = \frac{k}{(s+1)^3}$$

to get k: choose $s = 0$

$$\frac{V_2}{V_1}(s) = \frac{k}{1^3} = k = 1 \text{ as } V_2 = V_1 \text{ in the circuit}$$



as a check: let $s = 1$



$i_1 = 1 + 3/2 = 5/2$ amp

$\hat{v}_1 = 1 + 1/3 = 4/3$, $i_c = \frac{9}{8} \cdot \frac{4}{3} = 3/2$

$V_1 = 8/3 \cdot i_1 + \hat{v}_1 = 8/3 \cdot 5/2 + 4/3 = 48/6 = 8$. But $\frac{V_2}{V_1}(1) = \frac{k}{8} = 1/8 \Rightarrow k = 1$