Note Title 10/20/2019

ENEE 610
Th 10/24/19

$$E\partial x = Ax + Bu$$
$$y = Cx$$

$$y = C(E\partial - A)^{-1}B \cdot u, \quad T(\partial) = C(E\partial - A)^{-1}B$$

Ex: $y = \partial^3 u$

$$\begin{bmatrix} \partial & 0 & 0 & 0 \\ 0 & \partial & 0 & 0 \\ 0 & 0 & \partial & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [0, 0, 0, 1]x \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

1st row $x_2 = \partial x_1$

2nd row $x_3 = \partial x_2 = \partial^2 x_1$

3rd $x_4 = \partial x_3 = \partial^3 x_1$

4th $0 = -x_1 + u \quad \Longrightarrow \quad y = x_4 = \partial^3 x_1 = \partial^3 u$

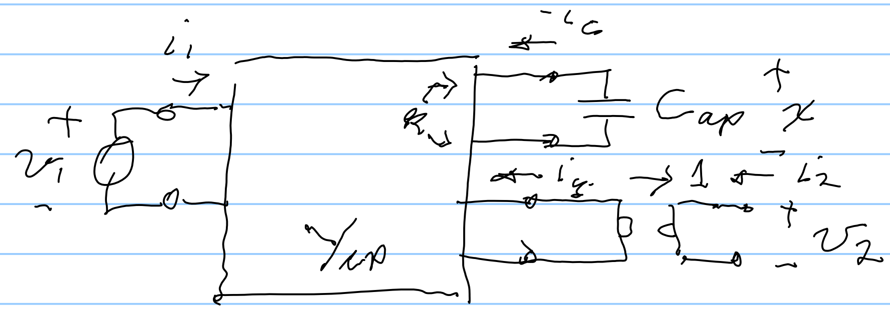
by extension can get any ∂^n, $n = 0, 1, \cdots \infty$

admittance synthesis from the state

$-C_{ap}\dot{x} = s1_k x = Ax + Bv_1$ $\qquad k$ = size of state

$v_2 = y = Cx + Dv_1$

set $y = i_g$ use the gyrator to change to a voltage



$C_{ap}\dot{x} = i_c$

(open circuit

$v_2 = g\, i_g$ $\qquad i_2 = -g v_1$

$\qquad\qquad\qquad\qquad\quad\;\; \| \qquad\quad \|$

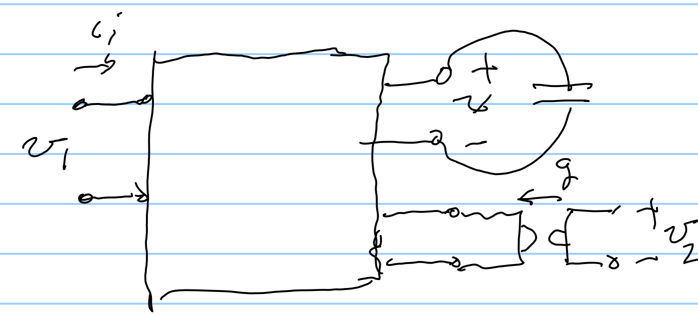
$\qquad\qquad\qquad\qquad\quad\;\; 0 \qquad\quad 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\quad$ a short

$$Y_{sp}\begin{bmatrix} v_1 \\ v_2 \\ x \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ -i_c \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & s1_k \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ x \end{bmatrix}$$

$$\begin{bmatrix} i_i \\ i_g \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ v_1 \end{bmatrix} = a 1 x$$

$$= \begin{bmatrix} & \hat{Y}_{cp} & \end{bmatrix}$$



$$\begin{bmatrix} i_1 \\ i_c \\ i_q \end{bmatrix} = \begin{bmatrix} \checkmark & \checkmark & \checkmark \\ B & A & 0 \\ D & C & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ x \\ v_2 \end{bmatrix} \qquad Y_{cp} = \begin{bmatrix} \checkmark & \checkmark & \checkmark \\ B & A & 0 \\ D & C & 0 \end{bmatrix}$$

choose the $\checkmark$'s to make Y_{cp} as skew as possible

$$Y_{cp} = \begin{bmatrix} 0 & -B^T & -D^T \\ B & A & 0 \\ D & C & 0 \end{bmatrix}$$

$\checkmark^T$ = transpose

Synthesize by single entry $y_{ij} \Rightarrow$ OTA

can transform the state $x = P\hat{x}$, P^{-1} exists

$$\alpha 1_k P\hat{x} = AP\hat{x} + Bu \quad , \quad y = CP\hat{x} + Du$$

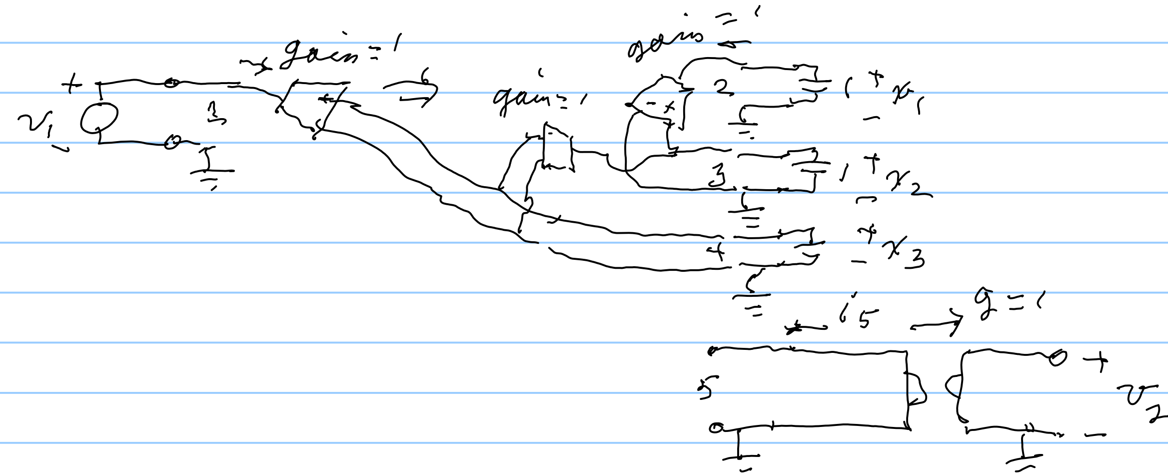
$$\alpha 1_k \hat{x} = P^{-1}AP\hat{x} + Bu \quad ; \quad Y_{cp} = \begin{bmatrix} 0 & -B^T & -D^T \\ B & P^{-1}AP & 0 \\ D & CP & 0 \end{bmatrix}$$

$$y = CP\hat{x} + Du$$

Ex:

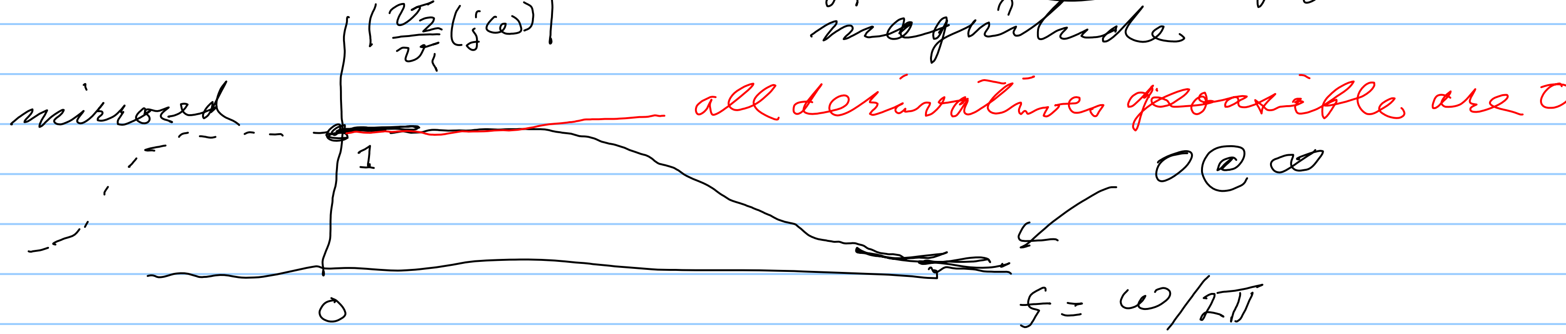
$$\alpha \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6, & -11, & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_1 , \quad v_2 = [1, 3, 1]x \qquad k = 3$$

$$
Y_{cp} = \left[\begin{array}{c|ccc|c}
0 & 0 & 0 & 1 & 0 \\
\hline
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & -6 & -11 & -6 & 0 \\
\hline
0 & 1 & 3 & 1 & 0
\end{array}\right]
$$



How to get transfer functions

Ex: desire a low-pass, maximally flat, v_2/v_1 magnitude



$$T(s) = \frac{v_2}{v_1}(s) = \frac{1}{1 + d_1 s + \cdots d_{\delta-1} s^{\delta-1} + s^{\delta}} \qquad \delta = \text{degree} = \delta\left[\frac{v_2}{v_1}(s)\right]$$

$$f(\omega) = |T(j\omega)|^2 = T(j\omega)\,T^*(j\omega) = T(j\omega)\,T(-j\omega) \quad \text{even in } \omega$$

$$\frac{d\,f(\omega)}{d\omega} = 0, \ldots, ; \qquad \frac{d\,1/f(\omega)}{d\omega} = -\frac{1}{f^2(\omega)} \cdot \frac{d\,f(\omega)}{d\omega} \quad \text{if } \frac{d\,f(\omega)}{d\omega} = 0$$

then so is $\frac{d(1/f(\omega))}{d\omega} = 0$

if $|f|^2 \neq 0$

$$\frac{1}{T \times T^*} = \frac{1}{f(\omega)} = \frac{1}{g(\omega^2)} \qquad g(\omega^2) = f(\omega)$$

$$= 1 + a_2 \omega^2 + \dots + a_{2\delta-2} \, \omega^{2\delta-2} + \omega^{2\delta}$$

set all $a_i = 0$