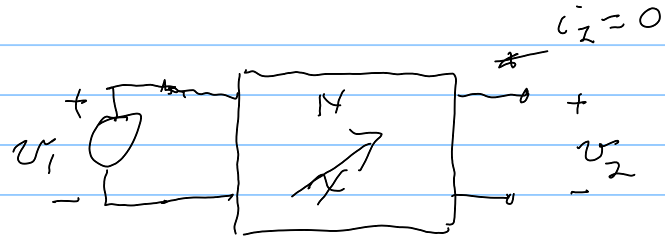
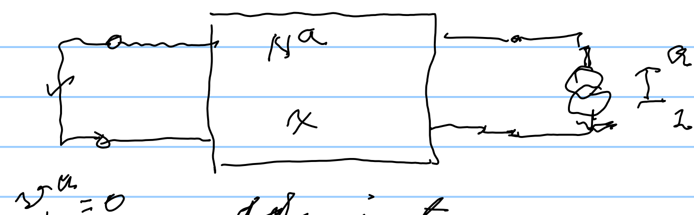
ENEE 610
Tu 10/22/19 $\Rightarrow$ 10/24/19

Note Title 10/20/2019

$S_x^{T(x)}$ = sensitivity of $T(x)$ [= transfer function] to the parameter x

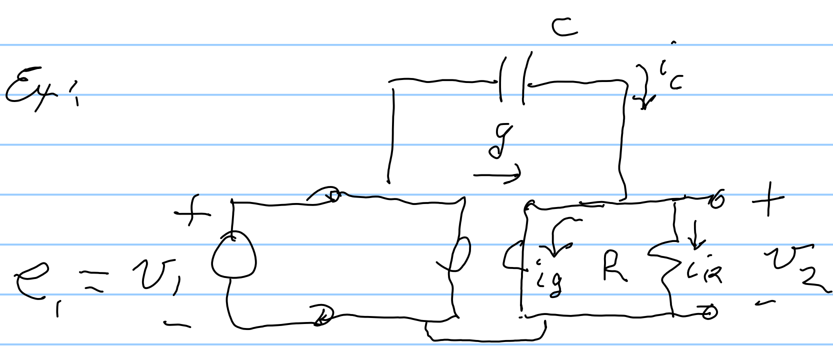
$$= \frac{1}{T(x)/x} \, dT(x)/dx = \frac{x}{T(x)} \, d(T(x))/dx$$





adjoint

Ex:



v_2/v_1 : KCL $0 = i_c - i_g - i_R$; $R = 1/G$

$$0 = (sC)(v_1 - v_2) - (-gv_1) - Gv_2$$

$$(sC + G)v_2 = (sC + g)v_1$$

$$T(x) = v_2/v_1\,(x) = \frac{xC+g}{xC+G} \qquad \frac{dT(x)}{dG} = (xC+g)\left[\frac{-1}{(xC+G)^2}\right] = \frac{-(xC+g)}{(xC+G)^2}$$

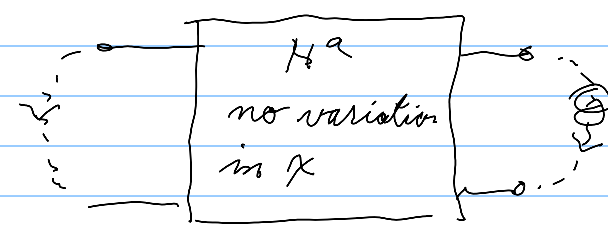
$$S_G^{T(x)} = \frac{G}{\left(\frac{xC+g}{xC+G}\right)} \cdot \frac{\left(-(xC+g)\right)}{(xC+G)^2} = \frac{-G}{(xC+G)}$$

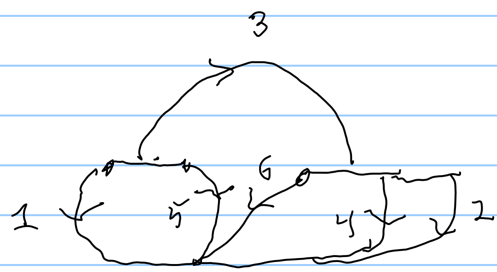
(note this is x dependent)

above is by direct calculation

By adjoint circuit

has same graph as the original





same for both N & N^a

$\therefore \quad v_b^{T} i_b = 0$ also $v_b^{a\,T} \cdot i_b = 0$, $v_b^{T} \cdot i_b^{a} = 0$

$$\Rightarrow \quad v_b^{T} i_b^{a} - v_b^{a\,T} i_b = 0$$

$$v_1 \, i_1^{a} + v_2 \, i_2^{a} - v_1^{a} i_1 - v_2^{a} i_2 + v_{2p}^{T} i_{2p}^{a} - v_{2b}^{a\,T} i_{2p} = 0$$

$v_1 = 0$: short; $i_2 = 0$: open; 2-port (v_{2p}); $= 0$: scalar

$$e_1 \cdot i_1^{a} + v_2 I_2^{a} - 0 - 0 + v_{2p}^{T} \cdot Y^{a} \cdot v_{2p}^{a} - v_{2p}^{a\,T} Y v_{2p} = 0$$

$v_{2p}^{a\,T} Y v_{2p} = v_{2p}^{T} \cdot Y^{T} v_{2p}^{a}$ — assumes existence of Y & Y^a

choose $e_1 \equiv 1\,v$

$$i_1^{a} + v_2 I_2^{a} + v_{2p}^{T} (Y^{a} - Y^{T}) v_{2p}^{a} = 0$$

$$\frac{d}{dx} : \quad \frac{d i_1^{a}}{dx} = 0 \; ; \quad \frac{d v_2}{dx} I_2^{a} + \frac{d v_{2p}^{T}}{dx} \left[Y^{a} - Y^{T} \right] v_{2p}^{a} + v_{2p}^{T} \left(\frac{dY^{a}}{dx} - \frac{dY^{T}}{dx} \right) \cdot v_{2p}^{a}$$

$$+ v_{2p}^{T} (Y^{a} - Y^{T}) \frac{d v_{2p}^{a}}{dx} = 0$$

$\frac{dY^{a}}{dx} = 0$

$\cancel{R} = 0$

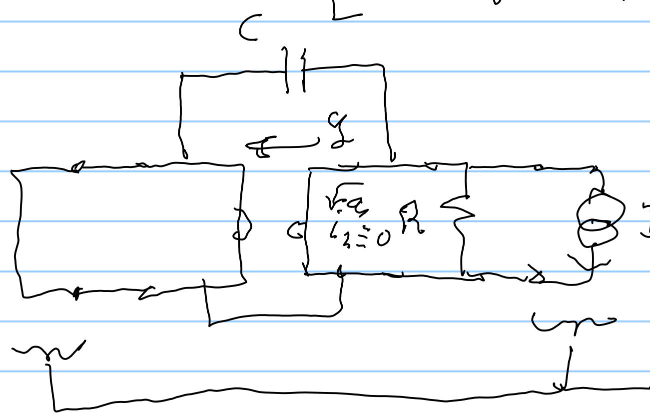
$$I_2^a \frac{d v_2}{dx} + \underbrace{\frac{d v_{2p}^T}{dx}\left[Y^a - Y^T\right]}_{\text{force to zero this matrix}} v_{2p}^a - v_{2p}^T \cdot \frac{dY}{dx} \cdot v_{2p}^a = 0$$

force to zero this matrix $\Big\}$ $Y^a = Y^T$

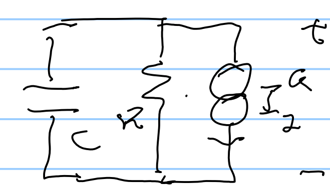
$$\therefore \frac{d^{\,v_2/v_1}}{dx} = \frac{1}{I_2^a}\left(v_{2p}^T \frac{dY}{dx} v_{2p}^a\right)$$

Here we can choose $Y = \begin{bmatrix} \alpha C & -\alpha C + g \\ -\alpha C - g & \alpha C + G \end{bmatrix} \Rightarrow Y^a = \begin{bmatrix} \alpha C & -\alpha C - g \\ -\alpha C + g & \alpha C + G \end{bmatrix}$

giving for N^a



I_2^a $+$ $v_{2p_2}^a$ $-$ load on N^a $=$



$i_2 = -I_2^a$

$$v_{2p_2}^a = \frac{-1}{\alpha C + G} I_2^a$$

$$= 2_{in}\, i_2$$

$$\frac{d^{\,v_2/v_1 = 1}}{dx} = \cancel{+} v_{2p}^T \frac{dY}{dx} \cdot v_{2p}^a \quad ; \quad Y = \begin{bmatrix} \alpha C & -\alpha C + g \\ -\alpha C - g & \alpha C + G \end{bmatrix}$$

$$x = G \quad , \quad \frac{dY}{dG} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_2^a \cdot \frac{d\left(\frac{v_2}{v_1}\right)}{dG} = + \begin{bmatrix} v_{2p_1} & v_{2p_2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{2p_1}^a \\ v_{2p_2}^a \end{bmatrix}$$

$$\frac{d\, v_2/v_1}{dY} = \frac{\cancel{-}1}{I_2^a} \cdot v_{2p_2} \cdot v_{2p_2}^a \quad ; \quad v_{2p_2} = v_2 = \frac{aC+g}{aC+G} \qquad v_2^a = \frac{-1}{aC+G} I_2^a$$

$$= + \frac{1}{I_2^a} \cdot \frac{aC+g}{aC+G} \cdot \frac{-1}{aC+G} \cdot I_{2a} \qquad \text{should agree}$$

$$S_G^{v_2/v_1} = S_x^T = \frac{G}{\frac{(aC+g)}{(aC+G)}} \cdot \frac{(aC+g)}{(aC+G)^2} \cdot (-1) = (-1) \cdot \frac{G}{aC+G}$$

$$\text{for } x = g \ , \quad \frac{dY}{dg} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \ , \quad \text{for } x = C \Rightarrow \frac{dY}{dC} = \begin{bmatrix} a & -a \\ -a & a \end{bmatrix}$$