Note Title 10/6/2019

ENEE 610 F 2019
Tu 10/10/19
updated 10/11/19

Hurwitz & strictly Hurwitz polynomials

|| no zeroes in $\sigma \geq 0$

no zeroes in $\sigma > 0$

simple on $j\omega$ axis $\doteq \sigma = 0$

if $p(s) = (s^2+\omega_0^2)\cdot p_2(s)$ then $(s^2+\omega_0^2)$ is common to $Ev[p(s)]$ & $Od[p(s)]$

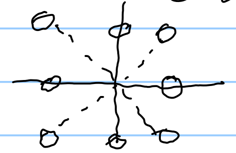
$\uparrow$ so will cancel in Od/Ev.

also any even polynomial will cancel

Positions of zeroes of an even polynomial $\frac{p(s)+p(-s)}{2} = Ev\,p(s)$

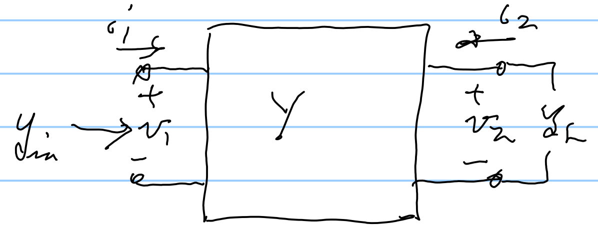
@ an even part zero of any polynomial with real coefficients

$2Ev[p] = 0 = p(s)+p(-s) = 0$ or $p(s) = -p(-s)$



zeroes of even part of $y(s)$ will cancel if k is a zero of $Ev[y(s)]$

for LPR choose any real $k>0$ then $(s-k)(s+k)$ cancel in the Richards function $\Rightarrow$ $\delta[R(s)] = [y(s)] - 1$



$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2, \quad i_2 = -y_L v_2$$

$$-y_L v_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow v_2 = -(y_L + y_{22})^{-1} y_{21} v_1$$

$$i_1 = y_{in} v_1 = y_{11} v_1 + y_{12} v_2 = [y_{11} - y_{12}(y_L + y_{22})^{-1} y_{21}] v_1$$

for a 2-port, all y_{ij} are scalars

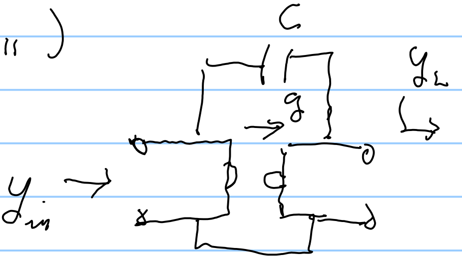
$$= \left[\frac{y_{11} y_{22} + y_{11} y_L - y_{21} y_{12}}{y_L + y_{22}}\right] = y_{in} = \frac{\Delta y + y_{11} y_L}{y_L + y_{22}}$$

$$\Delta = \text{determinant } Y = y_{11} y_{22} - y_{21} y_{12}$$

$$\Delta_y + y_{11} y_L = y_{in} \cdot y_L + y_{in} y_{22} \Rightarrow \left(y_{11} - y_{in}\right) y_L = y_{in} y_{22} - \Delta_y$$

$$y_L = \frac{(\Delta_y - y_{22} \cdot y_{in})}{(y_{in} - y_{11})}$$

Choose the 2-port



$$Y = \begin{bmatrix} Cs & -Cs+g \\ -Cs-g & Cs \end{bmatrix}$$

$$\Delta Y = (Cs)^2 - (-Cs+g)(-Cs-g) = (Cs)^2 - ((Cs)^2 - g^2) = g^2$$

$$y_L = \left(\frac{g^2 - Cs\, y_{in}(s)}{y_{in}(s) - Cs} \right)$$

$$= g^2 \left(\frac{\underline{1} - \frac{Cs}{g} \cdot \frac{y_{in}(s)}{g}}{g\left(\frac{y_{in}(s)}{g} - \frac{Cs}{g}\right)} \right)$$

$$R(s) = \left(\frac{k y(k) - s y(s)}{k y(s) - s y(k)} \right)$$

$$= \left(\frac{k y(k) \left(1 - \frac{s}{k} \cdot \frac{y(s)}{y(k)}\right)}{k y(k) \left(\frac{y(s)}{y(k)} - \frac{s}{k}\right)} \right)$$

$$y_L = g\left(\frac{1 - \frac{\lambda}{g/C}\cdot\frac{y_{in}(\lambda)}{g}}{\frac{y_{in}(\lambda)}{g} - \frac{\lambda}{g/C}}\right)$$

$$y(k)\,R(\lambda) = y(k)\left(\frac{1 - \frac{\lambda}{k}\cdot\frac{y(\lambda)}{y(k)}}{\frac{y(\lambda)}{y(k)} - \frac{\lambda}{k}}\right)$$

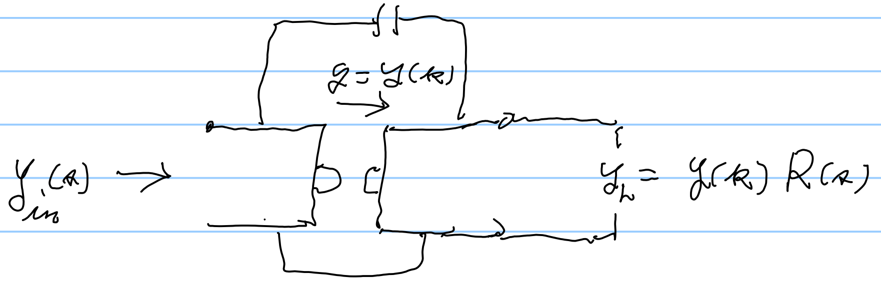
$$\shortparallel$$

$$y_L(\lambda)$$

let $y(k) = g$

$k = g/C \Rightarrow C = g/k = \frac{y(k)}{k}$

$C = y(k)/k$



choose k = a zero of $\mathrm{Ev}\left[y_{in}(\lambda)\right]$

then the degree of y_L is

lower than that of $y_{in}(\lambda)$

cancellation of $x+k$ if k is a zero of even $f(x)$, $f(x) = -f(-x)$ | @ $x = k, -k$

$k f(k) - x f(x)$; If $f(k) = -f(-k)$ Then This is $-k f(-k) - x f(x)$

@ $x = -k$, $-k f(-k) - (-k) f(-k) = 0$

Example: $y(x) = \dfrac{x(x^2+4)}{(x^2+1)}$, choose a k as $E_0[y] \equiv 0 \ \forall x$, $\delta[y] = 3$

choose $k = 2$, $y(2) = \dfrac{2(4+4)}{4+1} = \dfrac{16}{5}$; $g = \dfrac{16}{5}$, $C = \dfrac{y(2)}{2} = 8/5$

$$\frac{y_L}{16/5} = \left[\frac{k y(k) - x y(x)}{k y(x) - x y(k)}\right] = \left[\frac{\frac{32}{5} - x\left(\frac{x(x^2+4)}{x^2+1}\right)}{2\left(\frac{x(x^2+4)}{x^2+1}\right) - x\left(\frac{16}{5}\right)}\right] = \left[\frac{\left(\frac{32}{5}\right)x^2 + \frac{32}{5} - x^4 - 4x^2}{2x^3 + 8x - \frac{16}{5}x^3 - \frac{16}{5}x}\right]$$

$$\begin{array}{r} 40 \\ -16 \\ \hline 24 \end{array}$$

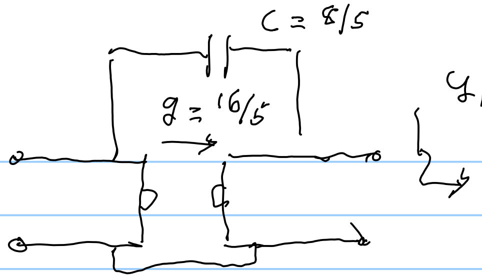
$$= \frac{x^4 - \frac{12}{5}x^2 - \frac{32}{5}}{\frac{6}{5}x^3 - \frac{24}{5}x}$$; $(x-2)(x+2)$ cancel so x^2-4 should divide out

$$\begin{array}{r|l} & x^2 + 8/5 \\ \hline x^2-4 & x^4 - \frac{12}{5}x^2 - \frac{32}{5} \\ & \underline{x^4 - 4x^2 - 32/5} \\ & \frac{8}{5}x^2. \end{array}$$

$$\begin{array}{r|l} & 6/5\, x \\ \hline x^2-4 & \frac{6}{5}x^3 - \frac{24}{5}x \\ & \underline{\frac{6}{5}x^3 - 24/5\, x} \\ & 0 \end{array}$$

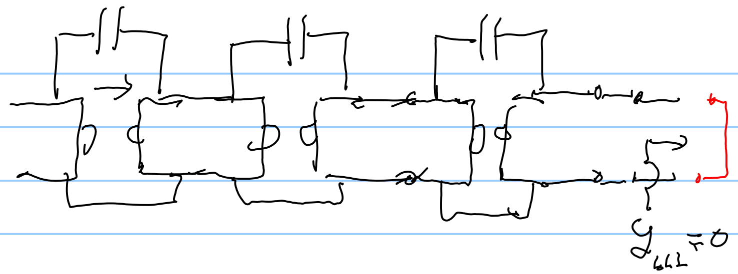
$$\frac{y_h(x)}{16/5} = \frac{(x^2-4)}{(x^2-4)} \frac{(x^2+8/5)}{6/5\, x} = \frac{x^2+8/5}{6x/5}$$

$\delta[y_h] = 2$



$$y_L = \frac{16}{5}\left(\frac{s^2+8/5}{6s/5}\right)$$

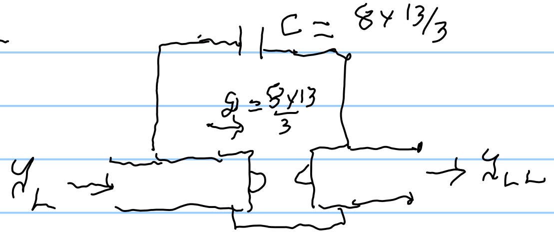
$\Rightarrow$



or $z = \infty$

added 10/11/19; details of last 2 stages:

choose $k = 1$, $y_L(1) = 8\times 13/3$

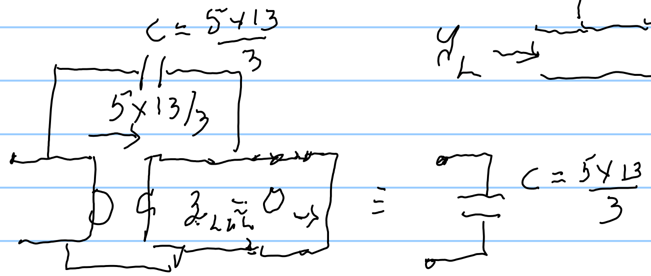


$$y_{LL}(s) = \frac{8\times 13}{3}\left(\frac{8\times 13}{3} - s\times\frac{16}{5}\left(\frac{5s^2+8}{6s}\right)\right) \Bigg/ \left(\frac{8}{3}\left(\frac{5s^2+8}{s}\right) - s\frac{8\times 13}{3}\right)$$

$$= \frac{8\times 13}{3}\left(\frac{-5s^3+5s}{-8s^2+8}\right) = \frac{5\times 13}{8}s$$

choose $k = 1$ again, $y_{LL}(1) = 5\times 13/3$

$$y_{LLL}(s) = \frac{5\times 13}{3}\left[\frac{\frac{5\times 13}{3} - \frac{5\times 13}{3}s^2}{\underbrace{\frac{5\times 13}{3}s - \frac{5\times 13}{3}s}_{0}}\right] \Rightarrow z_{LLL} \approx 0 \Rightarrow$$



note $z = 0$ is lossless

Next design via state equations of $v_2/v_1(s)$

$$\frac{v_2}{v_1} = \frac{\overset{=0}{n_2} s^2 + n_1 s + n_0}{\underset{\substack{\shortparallel\\1}}{d_2} s^2 + d_1 s + d_0}$$

$$s\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha_1 & -\alpha_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} v_1 \end{bmatrix}$$

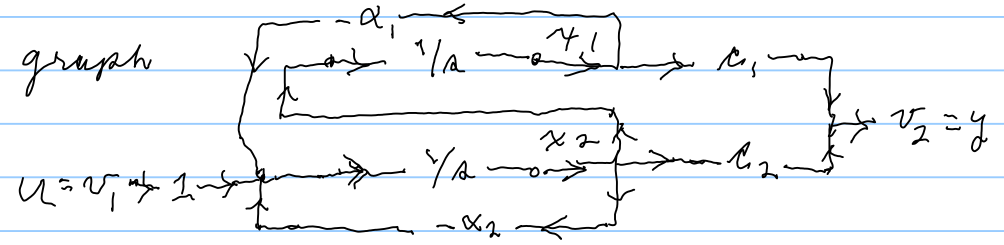
$$s1_k[x] = Ax + Bu$$
$$y = C\cdot x$$

$$y_2 = v_2 = \begin{bmatrix} c_1 & c_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$(+Du) = 0$ here

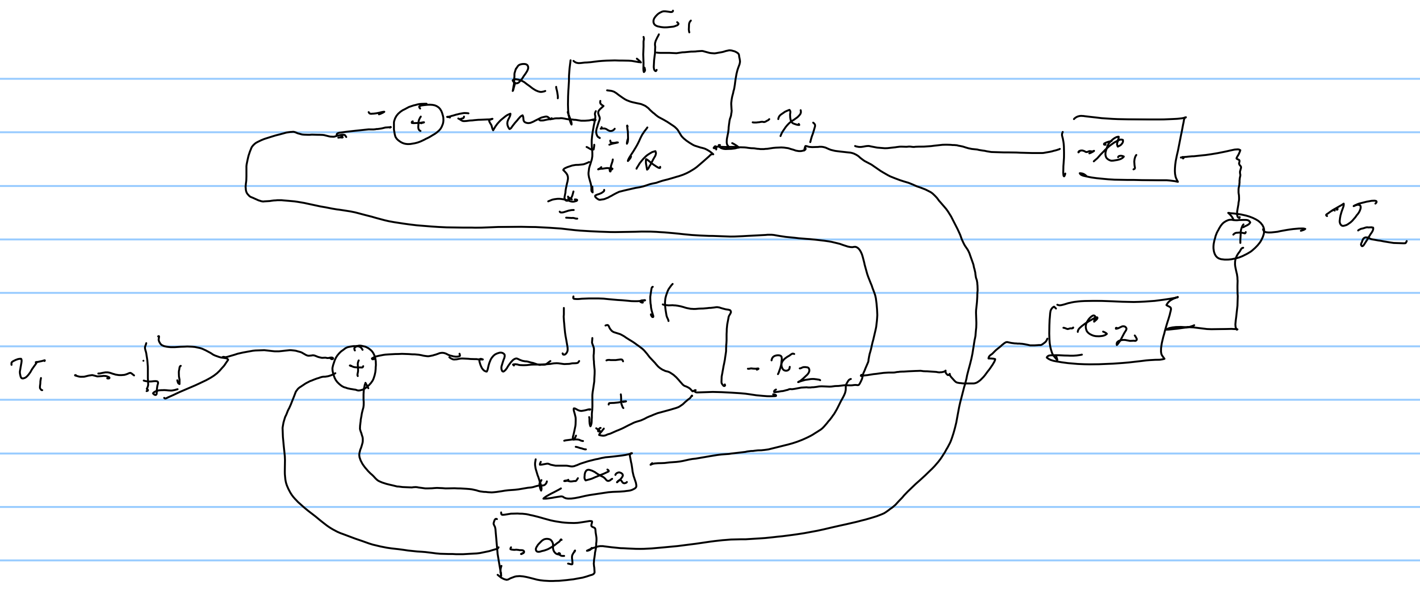
write as $x = \frac{1}{s}(Ax + Bu)$, $y = Cx$

signal flow graph



where $a \rightarrow \underset{\uparrow b}{\circ} \rightarrow c \equiv c = a + b$

$a \rightarrow f(s) \rightarrow b \equiv b = f(s)\cdot a$


v_1
R_1
C_1
$1/R$
$-x_1$
$-x_2$
$-\alpha_1$
$-\alpha_2$
$-c_1$
$-c_2$
v_2