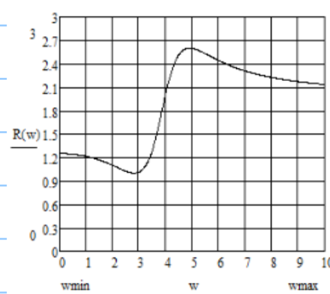
ENEE 610 F 2019
Tu 10/08/19

Note Title 10/6/2019

Example: z(s)=[2s^2+4s+20]/{s^2+2s+16] at s=j(sqrt(8)=jwo z(jwo)=1+j/sqrt(2)=Rmin+jX
so form Zm(s)=z(s)-Rmin = [s^2+2s+4]/[s^2+2s+16]. Then form Richards' function R(s)=[kZm(k)-sZm(s)]/[kZm(s)-sZm(k)]. Find k by forcing a zero of the denominator: jk/(sqrt(2)-jZm(k)sqrt(8): k=4{k^2+2k+4]/[k^2+2k+16] => k^3-2k^2+8k-16=(k-2)(K^2+8) or k=2; note k^2+wo^2 factors. Now Zm(k)=Zm(2)=[4+4+4]/[4+4+16]=12/24=1/2 so
R(s)=[1-sZm(s)]/[2Zm(s)-s/2] = 2[(s^2+2s+16)-(s^3+2s^2+4s)]/[(4s^2+8s+16)-(s^3+2s^2+16s)]
or R(s)=2[-s^3-s^2-2s+16]/[-s^3+2s^2-8s+16] = [(s-2)(s^2+3s+8)]/[(s-2)(s^2+8)]=1+3s/(s^2+8)

curve of R(jw)=Re[z(jw)]



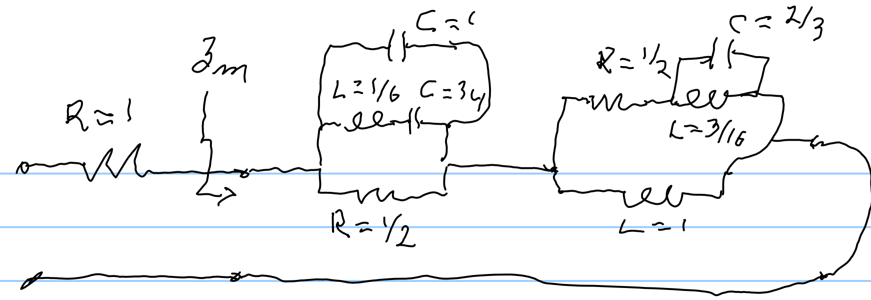
$$Z_m(s) = \frac{1}{\frac{R(s)}{Z_m(k)} + \frac{s}{k Z_m(k)}} + \frac{1}{\frac{k}{s Z_m(k)} + \frac{k}{s Z_m(k) R(s)}}$$

$$Z \rightarrow \simeq \frac{1}{\quad} + \frac{1}{\quad} + 0$$

$$y \rightarrow \left(2 + \frac{1}{\frac{s}{6} + \frac{4}{3s}}\right) + \frac{s}{1} \qquad \frac{1}{s} + \frac{1}{\left(\frac{1}{2} + \frac{1}{\frac{2s}{3} + \frac{16}{3s}}\right)}$$

$$R(s) = 1 + \frac{1}{\frac{s}{3} + \frac{8}{3s}}, \quad Z_m(k) = \tfrac{1}{2}, \quad k = 2$$

$Z(s) \longrightarrow$



Bott-Duffin synthesis

$$Z(s) = \frac{2s^2+4s+20}{s^2+2s+16} \text{ is PR}$$

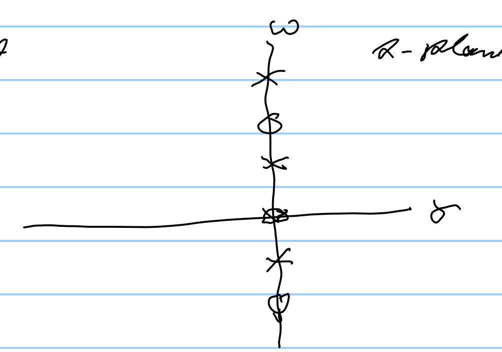
$\delta[Z(s)] = 2$, # of L, C's = 6

all passive

LPR $\quad y(s) = \frac{k_0}{s} + k_\infty \cdot s + \sum_{k_i} \frac{k_i s}{s^2+\omega_i^2}$, k_i are ≥ 0

(2nd Foster) for LPR 1-port

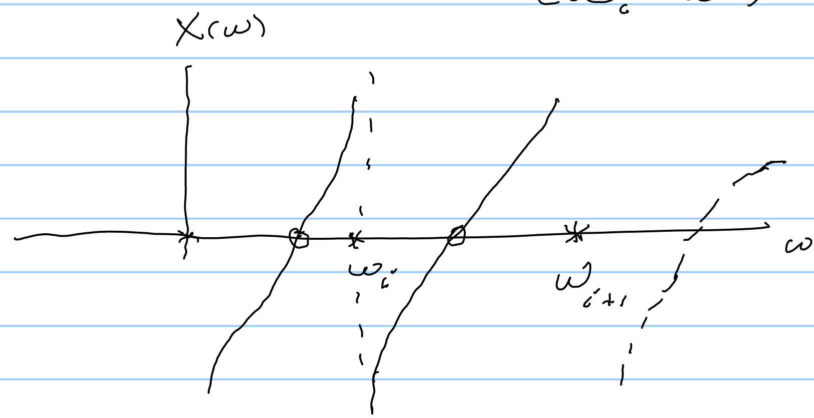
Poles & zeroes alternate on $j\omega$ axis



$$Y(j\omega) = \frac{-jk_0}{\omega} + jk_\infty\omega + \sum \frac{jk_i\omega}{-\omega^2+\omega_i^2} = jX(\omega)$$

$$\frac{dX(\omega)}{d\omega} = \frac{k_0}{\omega^2} + k_\infty + \sum \frac{k_i}{(-\omega^2+\omega_i^2)} - \frac{k_i\omega(-2\omega)}{(-\omega^2+\omega_i^2)^2} = \frac{k_0}{\omega^2} + k_\infty + \sum \frac{(-\omega^2k_i + k_i\omega_i^2 + 2\omega^2k_i)}{(\omega_i^2-\omega^2)^2}$$

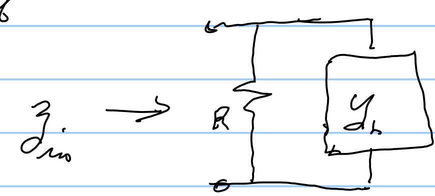
$$= \frac{k_0}{\omega^2} + k_\infty + \sum_i \frac{k_i(\omega^2+\omega_i^2)}{(\omega_i^2-\omega^2)^2} \geq 0 \qquad \infty > 0 \text{ @ pole } \omega_i$$



$\Longrightarrow$ poles & zeroes alternate

Ex: $y(s) = \dfrac{s(s^2+1)}{(s^2+1/2)(s^2+2)}$ is LPR but $y(s) = \dfrac{s(s^2+1/2)(s^2+2)}{(s^2+1)}$ is not LPR

Hurwitz test



y_L = load, LPR $\quad Z_{in} = \dfrac{1}{G + y_L(s)}$ is PR $G = 1/R > 0$

Let $y_L = \dfrac{N(s)}{D(s)}$, no common factors
N is Ev or Od
D is Od or Ev

$Z_{in} = \dfrac{1}{G + N/D} = \dfrac{D}{N + GD}$

no poles in $\sigma >$ or $j\omega$ axis

P(s) polynomial $= N(s) + G D(s)$ has no zeroes on $j\omega$ axis (if no common factors)

$\therefore$ P(s) is Hurwitz if $y(s)$ is LPR & $\delta[P] = \delta[y]$

form $y(s) = Od[P]/Ev[P]$ this is LPR if Hurwitz

Ex: $P(s) = s^5 + 5s^4 + 4s^3 + 2s^2 + s + 1$ form $\frac{Od}{Ev} = y(s) = \frac{s^5 + 4s^3 + s}{5s^4 + 2s^2 + 1}$

is $y(s)$ LPR try Cauer synthesis of $1/y$

$\frac{1}{5}s$

$5s^4 + 2s^2 + 1 \;\big)\; s^5 + 4s^3 + s$

$s^5 \quad \frac{2}{5}s^3 + \frac{1}{5}s$

$\frac{18}{5}s^3 + \frac{4}{5}s$

$\frac{25}{18}s$

$\frac{18}{5}s^3 + \frac{4}{5}s \;\big)\; 5s^4 + 2s^2 + 1$

$5s^4 + \frac{20}{18}s^2$

$\frac{16}{18}s^2 + 1$

$\frac{18^2}{16\times5}s$

$\frac{16}{18}s^2 + 1 \;\big)\; \frac{18}{5}s^3 + \frac{4}{5}s$

$\frac{18}{5}s^3 + \frac{18^2}{16\times5}s$

$\frac{(64-324)}{16\times5}s$

$-\frac{52}{16}$

$(-\#)s \;\big)\; \frac{16}{18}s^2 + 1 \quad -\frac{52}{16}s$

1

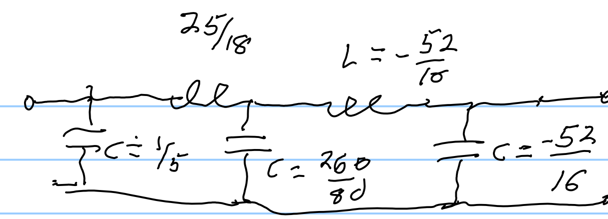
$16 \times 4 = 64$

$18 \times 18 = 144$, 18, 324

$324 - 64 = 260$

$\frac{52}{\cancel{260}} \Big/ \cancel{5\times}16$

$$y(\alpha) = \frac{1}{5}\alpha + \cfrac{1}{\frac{25}{18}\alpha + \cfrac{1}{\frac{260}{80}\alpha + \cfrac{1}{-\frac{52}{16}\alpha + \cfrac{1}{-\frac{52}{16}\alpha}}}}$$



due to a negative coefficient $P(\alpha)$ is not Hurwitz