ENEE 610
Th 10/03/19

Note Title 9/29/2019

$$R(\lambda) = \frac{k f(k) - \lambda f(\lambda)}{k f(\lambda) - \lambda f(k)} = \frac{N_R}{D_R}$$

if $k > 0$ is real and $f(\lambda)$ is PR
then $R(\lambda)$ is PR
$\delta[R(\lambda)] = \delta[f(\lambda)]$ or $\delta[f(\lambda)] - 1$ if k is a zero of $Ev[f(\lambda)]$

$$Ev[f(\lambda)] = \frac{f(\lambda) + f(-\lambda)}{2}, \quad Od[f(\lambda)] = \frac{f(\lambda) - f(-\lambda)}{2}$$

if $f(\lambda)$ is LPR, $Ev[f(\lambda)] \equiv 0$

$f(\lambda)$ is like $Z(\lambda)$ or $Y(\lambda)$

Check $S(\lambda)$ = reflection coefficient (scattering for a $n=1$-port)

$$= \frac{1 - y(\lambda)}{1 + y(\lambda)} \Longrightarrow S_R(\lambda) = \frac{1 - R(\lambda)}{1 + R(\lambda)} = \frac{[k f(\lambda) - \lambda f(k)] - [k f(k) - \lambda f(\lambda)]}{[k f(\lambda) - \lambda f(k)] + [k f(k) - \lambda f(\lambda)]} = \frac{1 - \frac{N_R}{D_R}}{1 + N_R/D_R} = \frac{D_R - N_R}{D_R + N_R}$$

$$= \frac{k[f(\lambda) - f(k)] + \lambda[-f(k) + f(\lambda)]}{k[f(\lambda) + f(k)] + \lambda[-f(k) - f(\lambda)]} = \frac{(k+\lambda)[f(\lambda) - f(k)]}{(k-\lambda)[f(\lambda) + f(k)]}$$

$\uparrow$ pole at $k = s$ cancels with zero of $f(s) - f(k)$

$|S_k(j\omega)| \leq 1$ for BR = bounded real, rational

$$= \left|\frac{k + j\omega}{k - j\omega}\right| \cdot \Bigg|\underbrace{\frac{\frac{f(s)}{f(k)} - 1}{\frac{f(s)}{f(k)} + 1}}_{BR}\Bigg| = +\sqrt{\frac{k^2+\omega^2}{k^2+\omega^2}} = 1$$

now $\frac{f(s)}{f(k)}$ is like $Z(s)$

as $f(s)/f(k)$ is PR

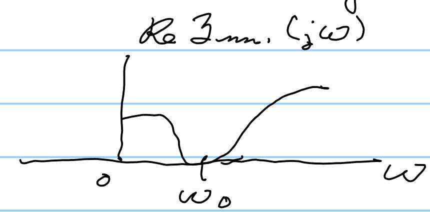
Bott-Duffin synthesis, given a PR $Z(s)$

1) subtract out any $j\omega$ axis poles $k_\infty s$, k_0/s or $\frac{k_i s}{s^2+\omega_i^2}$, $k_i \geq 0$

2) $\mathrm{Re}\, Z(j\omega) \geq 0$, remove a resistor to get it zero gives $Z_{minimum}$, $\mathrm{Re}\, Z_{min}(j\omega_0) = 0$

$R_m = \min \mathrm{Re}\, Z(j\omega)$

$Z(s) - R_m = Z_{min}$



, $\mathrm{Im}\, Z(j\omega_0)$ can be + or −

3) From the Richards' function for $z_{min} = z_m$

$$R(a) = \left[k z_m(k) - a z_m(a)\right] \Big/ \left[k z_m(a) - a z_m(k)\right]$$

assume know $k > 0$ solve for z_m as a function of $R(a)$

$$\left(k z_m(a) - a z_m(k)\right) R = k z_m(k) - a z_m(a)$$

$$k z_m(a) \cdot R + a z_m(a) = k z_m(k) + a z_m(k) R$$

$$z_m(a) = \frac{k z_m(k) + a z_m(k) R(a)}{k R(a) + a} = \frac{k z_m(k)}{k R(a) + a} + \frac{a z_m(k) R(a)}{k R(a) + a}$$

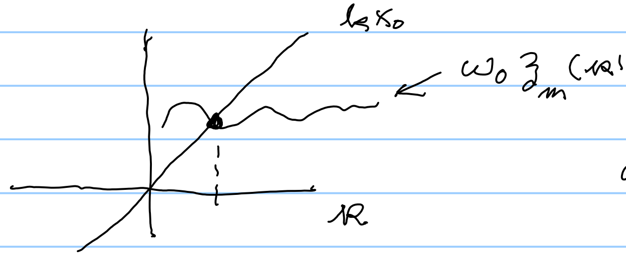
$$= \frac{1}{\frac{R(a)}{z_m(k)} + \frac{a}{k z_m(k)}} + \frac{1}{\frac{k}{a z_m(k)} + \frac{a}{a z_m(k) R(a)}}$$

Now choose k to give $R(s)$ a pole on $j\omega$ axis; when remove the pole then $\delta[R] = \delta[Z_{in}] - 2$

Look @ $R(j\omega_0) = \dfrac{k Z_{in}(k) - j\omega_0 Z_{in}(j\omega_0)}{k Z_{in}(j\omega_0) - j\omega_0 Z_{in}(k)}$ note $Z_{in}(j\omega_0) = jX_0$ X_0 can be >0 or <0

$= \dfrac{k Z_{in}(k) + \omega_0 X_0}{j(k X_0 - \omega_0 Z_{in}(k))}$ if $X_0 > 0$ then force denominator to 0 by $kX_0 - \omega_0 Z_{in}(k)$

desire to find $k = \dfrac{\omega_0 Z_{in}(k)}{X_0}$



a solution $k>0$ exists as $\omega_0 Z_{in}(k)$ is bounded & k is unbounded linear

this k forces $R(s)$ to have its denominator 0 @ $s = j\omega_0 \Rightarrow j\omega$ axis pole of a PR function

$\therefore$ can decrease degree of $R(s)$ by a passive circuit

$$R(s) = \frac{k_0 s}{s^2+\omega_0^2} + Z_{remainder}(s),$$

$\uparrow$ PR & $\delta[Z_{remainder}] = \delta[R] - 2$

$$Z_{LC} = \frac{k_0 s}{s^2+\omega_0^2} = \frac{1}{\frac{s}{k_0} + \frac{\omega_0^2}{k_0 s}}$$